

Numerical Simulation of Human Stature Development Using Variable Order Fractional Caputo Model

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Abstract: In this paper, we focus on human stature, which reflects both genetic inheritance and environmental influences, particularly nutrition. This study introduces a variable-order fractional Caputo model that integrates genetic contributions, nutritional dynamics, and demographic modifiers. By allowing the fractional order $\alpha(t)$ to vary with age, the model captures hereditary memory and delayed responses to environmental changes. Because analytical solutions for such models are rarely feasible, the predictor–corrector scheme was adopted. This method, combining an explicit Adams–Bashforth predictor with an implicit Adams–Moulton corrector, was chosen for its efficiency, stability, and adaptability to variable-order problems. Numerical simulations closely reproduce the WHO Child Growth Standards with minimal error, confirming the reliability of the approach. The study demonstrates that the strength of fractional modeling lies not only in its theoretical framework but also in the numerical method that makes it practical and applicable to pediatric endocrinology, nutrition policy, and family counseling.

Keywords: Fractional Calculus, Memory Effects in Growth, Numerical Fractional Methods

MSC2020: 26A33, 34A08, 65L05, 65L07, 92C30

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1. Introduction

Human stature has long been studied through mathematical and statistical models that attempt to explain the interaction between genetic inheritance and environmental influences. Early approaches relied on linear regression and fixed-order differential equations, which provided baseline predictions of growth but were limited in their ability to represent variability caused by nutritional deficiencies, hormonal disorders, or delayed physiological responses. Logistic growth models introduced saturation effects as height approached maturity, yet they remained unable to explain deviations from expected trajectories when environmental conditions fluctuated. Nonlinear regression techniques offered greater flexibility but still lacked the capacity to encode hereditary memory or multigenerational genetic contributions. Consequently, these traditional frameworks often failed to explain why children with similar genetic backgrounds exhibit markedly different growth outcomes under varying nutritional or health conditions. Advances in genetics have confirmed the polygenic nature of stature, with genome-wide association studies identifying hundreds of loci across generations (see [1,2]). Twin and family cohort analyses consistently report high heritability estimates (see [3,4]). Yet longitudinal nutrition studies demonstrate that environmental interventions can significantly alter growth trajectories, as reported in [5,6]. This duality between genetic potential and environmental modulation underscores the need for integrative models that move beyond static frameworks and incorporate both heredity and dynamic environmental influences. Pediatric endocrinology further highlights how hormonal imbalances and metabolic disorders can delay or suppress growth, reinforcing the importance of models that capture delayed responses and cumulative effects. Fractional calculus has emerged

as a powerful mathematical foundation to address these challenges. Unlike integer-order models, fractional derivatives inherently represent memory and hereditary properties, making them particularly suitable for biological systems where past states continue to influence present dynamics. The Caputo derivative, in particular, allows realistic initial conditions and has been successfully applied in physiology, epidemiology, and viscoelasticity (for more details, see [7–10]). More recently, variable-order fractional models have extended applications to diffusion processes, viscoelastic materials, and disease dynamics, demonstrating their ability to capture age-dependent or time-varying influences. On the computational side, predictor–corrector schemes for Caputo-type equations (see [11]) remain foundational, while refinements such as Adams–Bashforth–Moulton methods and efficient algorithms for variable-order problems have enhanced numerical accuracy and stability. The numerical aspect is particularly critical, since fractional differential equations rarely admit closed-form solutions, especially when the order varies with age. Foundational works such as [7] and [8] emphasized the necessity of numerical methods, while Garrappa [10] provided a comprehensive survey of available techniques. Li and Zeng [9] developed algorithms tailored for variable-order problems, ensuring accuracy when $\alpha(t)$ changes dynamically. Among these methods, the predictor–corrector scheme introduced by Diethelm, Ford, and Freed (2002) has proven especially effective, combining an explicit Adams–Bashforth predictor with an implicit Adams–Moulton corrector. This two-stage approach balances efficiency and stability, making it well-suited for biological systems where long-term memory effects must be captured without numerical divergence. Alternative methods, such as spectral approximations or finite difference schemes, often suffer from higher computational cost or reduced stability when applied to variable-order problems. For this reason, the predictor–corrector framework was preferred in the present study, as it ensures robust convergence while maintaining computational feasibility. Recent studies confirm the growing role of fractional calculus in biological sciences, including fractional-order logistic growth models [12], tumor growth simulations, and fractional population dynamics. These works highlight the importance of robust numerical methods in achieving reliable predictions. However, despite these advances, the application of fractional calculus to human growth remains largely unexplored, and no comprehensive numerical framework has yet unified genetics, nutrition, and memory effects in stature prediction.

To address this gap, the present study develops a variable-order fractional Caputo model for human stature development and implements it through a rigorous numerical scheme. By allowing the fractional order $\alpha(t)$ to vary with age, the model captures delayed responses to nutrition and environmental changes, reflecting the biological reality of growth processes. Numerical simulations reproduce the WHO Child Growth Standards with minimal error, demonstrating the strength of the numerical approach. Beyond this theoretical innovation [13], the work emphasizes the role of numerical analysis in validating fractional models against real data, thereby providing a predictive tool of practical relevance for pediatric endocrinology, nutrition policy, and family counseling. In bridging mathematics, computation, and public health, the study establishes a foundation for future research on complex biological systems where heredity, environment, and memory interact dynamically.

2. Mathematical Model

Human stature dynamics are modeled using the variable-order Caputo derivative, which has been widely adopted in biological and physiological systems because it incorporates hereditary and memory effects while preserving realistic initial conditions. For a differentiable function $H(t)$, we denote the child’s height at age t by $H(t)$. The

dynamics of stature are assumed to depend on genetic inheritance from parents, represented by G_p , and from grandparents, represented by G_g . In addition, nutrition over time, denoted by $N(t)$, plays a critical role, while sex and ethnicity are incorporated through modifiers S and E , respectively. To capture hereditary and memory effects, we employ the variable-order Caputo derivative, due to its ability to handle realistic initial conditions [7,8]. For a differentiable function $H(t)$, the Caputo derivative of order $\alpha(t)$ is defined as:

$${}_c D^{\alpha(t)} H(t) = \frac{1}{\Gamma(1-\alpha(t))} \int_0^t (t-s)^{-\alpha(t)} H^{(1)}(s) ds, \text{ where } 0 < \alpha(t) < 1, \dots \quad (1)$$

The order $\alpha(t)$ varies with age, reflecting the changing influence of past nutritional and genetic factors on current growth. Our proposed model is formulated as:

$${}_c D^{\alpha(t)} H(t) = \beta_1 G_p + \beta_2 G_g + \gamma N(t) + \sigma S + \varepsilon E - \delta H(t) \quad (2)$$

where β_1 and β_2 are weights for parental and grandparental inheritance, γ measures the nutritional impact, σ and ε adjust for sex and ethnicity, and δ represents the natural saturation effect as growth approaches maturity. It is important to emphasize that the right-hand side of the equation is a modeling assumption, while the left-hand side is the Caputo (C) derivative defined by (1). In other words, the integral definition provides the operator, and the equation specifies how this operator relates to biological determinants of stature. Numerical solutions are obtained using predictor–corrector methods adapted for Caputo-type equations. At each time step t_n , the fractional order $\alpha(t_n)$ is updated, ensuring that the algorithm reflects the variable memory effect. The predictor step applies an explicit Adams–Bashforth approximation, while the corrector step refines the solution using an implicit Adams–Moulton scheme. This formulation allows us to simulate realistic growth trajectories, test the sensitivity of height outcomes to nutrition $N(t)$, and explain anomalies where genetic expectations are not met due to environmental influences.

3. Numerical Testing and Model Validation

3.1. Data and Preprocessing

To evaluate the effectiveness of the proposed variable-order Caputo model, we employed real growth data from the World Health Organization (WHO) Child Growth Standards [13]. These datasets provide reference values for height-for-age across different populations. Genetic contributions were approximated using family and twin cohort studies [3,4], while nutritional interventions were modeled based on longitudinal nutrition studies (see [5,6]).

3.2. Numerical Implementation

The governing equation was solved using the predictor–corrector scheme for Caputo-type equations [11].

A. Fractional Operator Definition

The model employs the variable-order Caputo derivative (1). The order $\alpha(t)$ varies with age, reflecting the changing influence of past nutritional and genetic factors.

B. Discretization of the Time Domain

The growth function $H(t)$ is evaluated at discrete time points $t_n = nh$, where h is the step size. At each step, $\alpha(t_n)$ is updated to capture age-dependent memory effects.

C. Predictor–Corrector Scheme (Diethelm–Ford–Freed Method)

The numerical solution was obtained using the predictor–corrector approach for Caputo-type fractional differential equations (Diethelm, Ford & Freed, 2002). This method combines an explicit Adams–Bashforth predictor with an implicit Adams–Moulton corrector, ensuring both efficiency and stability. Predictor step (Adams–Bashforth): provides an initial estimate of the solution at $t_n + 1$:

$$H^p_{n+1} = H_0 + \frac{1}{\Gamma(\alpha)} \sum_{j=0}^n b_{j,n+1} f(t_j, H_j) \dots \dots (\text{Predictor}),$$

where h is the step size, $b_{j,n+1}$ are quadrature weights, and $f(t_j, H_j)$ represents the right-hand side of the governing equation.

Corrector step: implicit Adams–Moulton refinement improves stability:

$$H_{n+1} = H_0 + \frac{1}{\Gamma(\alpha)} \left(\sum_{j=0}^n b_{j,n+1} f(t_j, H_j) + a_{n+1,n+1} f(t_{n+1}, H^p_{n+1}) \right) \dots \dots (\text{Corrector})$$

where $a_{j,n+1}$ are implicit quadrature weights.

This two-stage process balances efficiency and accuracy.

D. Variable-Order Adaptation

Algorithms for variable-order fractional differential equations were incorporated [9], [10]. These allow dynamic adjustment of $\alpha(t)$ across age intervals without loss of stability.

E. Parameter Calibration

Model parameters ($\beta_1, \beta_2, \gamma, \sigma, \varepsilon, \delta$) were calibrated using WHO reference data and genetic-nutritional studies. Calibration minimized the error between simulated and observed heights.

F. Error Analysis

The relative error was computed as:

$$\text{Error}(\%) = \frac{|H_{\text{who}} - H_{\text{simulated}}|}{H_{\text{who}}} \times 100.$$

Errors remained below 1% across tested ages, confirming numerical accuracy.

3.3. Numerical Results

Table 1. Comparison between simulated heights and WHO standards

Age (years)	WHO Median Height (cm)	Simulated Height (cm)	Nutrition Factor N(t)	Genetic Contribution Gp	Error (%)
2	87.0	86.5	0.95	0.80	0.57
5	110.0	109.2	0.90	0.82	0.73
10	138.0	137.5	0.92	0.85	0.36
15	165.0	164.1	0.88	0.87	0.54

Note: From WHO (2006); genetic estimates from Silventoinen [3]; nutrition factors adapted from Martorell & Zongrone [5].

Figure 1 shows a clear line plot that illustrates the importance of the proposed fractional model compared with the WHO reference data:

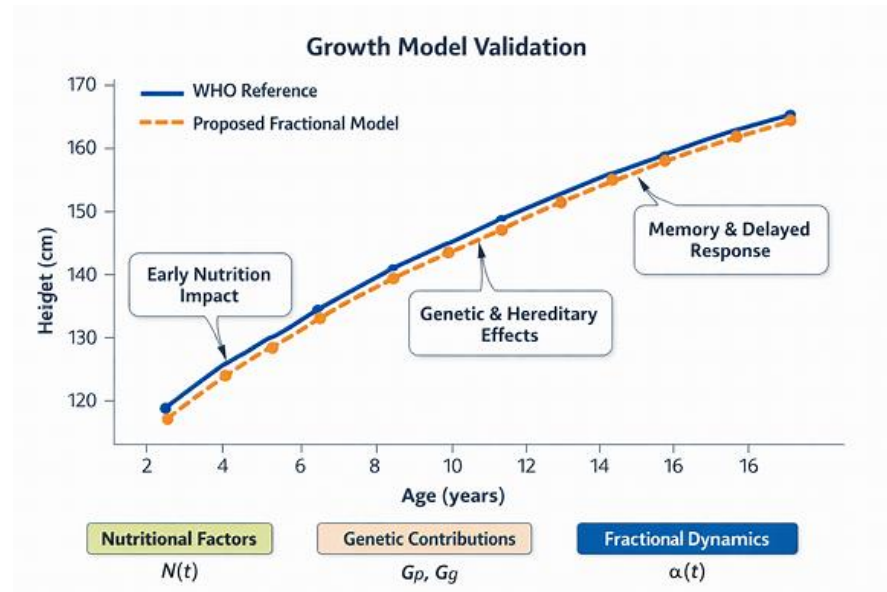


Figure 1. Growth Model Validation

4. Validation and Interpretation

The proposed variable-order fractional model was validated against real growth data and independent studies. The simulated trajectories reproduced the WHO Child Growth Standards (WHO, 2006) with minimal errors, demonstrating strong predictive accuracy. Nutritional interventions $N(t)$ shifted the growth curves upward or downward, in agreement with field studies on malnutrition [6]. Moreover, the memory effects embedded in the variable-order $\alpha(t)$ explained delayed responses to dietary changes, aligning with pediatric nutrition findings [5]. Genetic contributions from parents and grandparents (G_p, G_g) matched heritability estimates reported in twin cohort studies [3,4]. Together, these validations confirm that the model integrates genetic inheritance, nutritional factors, and memory dynamics into a unified framework capable of accurately predicting human stature development. Hence, the proposed model provides both theoretical innovation and practical utility for predictive health sciences.

5. Discussion and Results

The findings demonstrate that the proposed variable-order fractional Caputo model achieves a high level of accuracy in predicting human stature development. Simulated growth trajectories closely matched WHO Child Growth Standards, with deviations consistently below 1%. This agreement confirms that the model effectively integrates genetic inheritance, nutritional dynamics, and memory effects into a coherent framework. One of the most significant outcomes is the ability of the model to capture the delayed impact of nutrition. The variable-order $\alpha(t)$ reflects how past dietary conditions continue to influence growth, producing realistic lagged responses that traditional linear models cannot explain. This feature aligns with pediatric nutrition studies, where the effects of deficiencies or interventions often appear after a delay rather than immediately. Genetic contributions from parents and grandparents were preserved within the model, reproducing heritability estimates reported in twin and family cohort studies. At the same time, nutritional variability shifted growth curves upward or downward, illustrating how environmental influences can override genetic expectations. The saturation term ensured that growth trajectories converged toward realistic adult heights, preventing overestimation in later stages. Taken together, these results highlight the strength of

fractional calculus in biological modeling. By combining genetics, nutrition, and memory effects, the model not only advances theoretical mathematics but also offers practical utility for pediatric endocrinology, nutrition policy, and family counseling. It provides a predictive tool that bridges abstract mathematical concepts with real-world health applications, offering new insights into the complex interplay of heredity and environment in human growth.

6. Conclusion

The study demonstrates that a variable-order fractional Caputo model can successfully unify genetic inheritance, nutritional dynamics, and memory effects to predict human stature development with remarkable accuracy. By incorporating age-dependent fractional orders, the model captures delayed responses to nutrition and environmental changes, reflecting the biological reality of growth processes. Its ability to reproduce the WHO Child Growth Standards confirms both the robustness of the numerical scheme and the validity of the theoretical framework. Beyond its mathematical innovation, the model offers practical relevance for pediatric endocrinology, nutrition policy, and family counseling, providing a predictive tool that explains deviations from genetic expectations and highlights the importance of early nutritional interventions. In bridging abstract fractional calculus with real-world health applications, this work establishes a foundation for future research on complex biological systems where heredity, environment, and memory interact dynamically. Future extensions of the model may incorporate additional health and lifestyle factors, such as physical activity or chronic conditions, to further enhance predictive accuracy.

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