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Mathematical Modeling of the Price Volatility of Maize and Sorghum between 1960 and 2022

Samuel Olorunfemi Adams^{1,*}, Mohammed Anono Zubair¹, Michael Franklin Ezike¹

¹ Department of Statistics, University of Abuja, Abuja, Nigeria

*Correspondence: Samuel Olorunfemi Adams (samuel.adams@uniabuja.edu.ng)

Abstract: The price of grains like maize and sorghum is subject to significant fluctuations, which can have a significant impact on a country's economy and food security. The aim of the study is to model sorghum and maize price volatility in Nigeria. The data utilized in the study was extracted from World Bank Commodity Price Data (WBCPD), 2022. The data consists of monthly prices in nominal US dollars for maize and sorghum from January 1960 – August 2022. The Autoregressive Conditional Heteroskedasticity (ARCH) and Generalized Autoregressive Conditional Heteroskedasticity (GARCH) models were utilized for capturing the two-grain price volatility. Two types of conditional heteroscedastic models exist, the first group uses exact functions to control the evolution of σ_t^2 , while the second group describes σ_t^2 with stochastic equations. It is inferred from the result that inherent uncertainties and fluctuations existed in the prices of maize and sorghum in Nigeria which implies that the price volatility is positive and statistically significant suggesting that historical information and past shocks play a crucial role in determining the volatility observed in the grains. It is recommended that the ARCH, GARCH, EGARCH, TGARCH, PARCH, CGARCH, and IGARCH models should be employed for modeling and managing the volatility of maize and sorghum prices in Nigeria. These models have shown effectiveness in capturing different aspects of volatility, including the impact of past shocks, conditional volatility, asymmetry, and other relevant factors.

Keywords: ANN, ARCH, GARCH, Maize, Sorghum, AIC, Volatility

How to cite this paper:

Adams, S. O., Zubair, M. A., & Ezike, M. F. (2023). Mathematical Modeling of the Price Volatility of Maize and Sorghum between 1960 and 2022. *Journal of Mathematics Letters*, 1(1), 38–56. Retrieved from <https://www.scipublications.com/journal/index.php/jml/article/view/801>

Academic Editor:

Mohammad Alqudah

Received: September 16, 2023

Revised: October 29, 2023

Accepted: November 1, 2023

Published: November 3, 2023



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1. Introduction

Sorghum and maize are among the most important staple foods in poor rural communities in the semi-arid tropics of Africa and Asia. They ranked third and fourth most important cereal after wheat and rice. Sorghum is known by several names such as guinea corn, Egyptian millet, Sudan grass, great millet, and other native names depending on the locality. It belongs to the grass family, often called sorghum bicolor [1]. Apart from being a source of food, Sorghum is also used for fiber, fodder, and the production of alcoholic beverages. It grows well in the tropical and subtropical regions of Africa and Asia. Sorghum grain has various shapes, sizes, and colors. Some have tight-headed, round, open, and droopy panicles [2]. The colors include red, orange or bronze, white, tan, and black. The red, orange, and bronze strains are traditionally grown and used in all segments of the industry, while the tan, cream, and white types are processed into flour for the food industry. The black strain has anti-oxidant properties and is used in other food varieties, it is also a major source of energy, protein, vitamins, and minerals [3].

Maize is one of the most important grains in the world. The crop is consumed as a staple food in Nigeria, and some other tropic Africa counties, it accounts for about 43% of calories in the diet of an average Nigerian [4]. Several studies on maize production have pointed out that the crop increased across all agro-ecological zones of the country. The

crop has been utilized by food processing industries, pharmaceutical, herbal and medicinal sectors. The crop was reported by [5] to be used as a local 'cash crop', indicating that 30% of the land has been devoted to maize cultivation. An increase in maize production to 1 – 3 hectares in any farming system was reported by [6] to be able to combat hunger in a household, in addition to increasing food production, especially in Africa. An increase in maize production from 612 thousand tonnes to 70195 thousand tonnes has been reported by [7], representing a 100% increase in production. About 561,397.29 hectares of arable land in Nigeria has been put into maize production with an increase in the crop price, pointing to the importance of maize in the country's economy.

Grains such as maize and sorghum are staple crops in Nigeria and fluctuations in their prices can have a significant impact on the country's economy and food security. Modeling and predicting the volatility of grain prices can help farmers make more informed decisions about planting and harvesting. Traders make better decisions about buying and selling, and policymakers develop more effective policies to promote food security and stability in the agricultural sector. In recent years, there have been several studies that have used econometric models to analyze the volatility of grain prices in Nigeria. These studies have applied models such as GARCH (Generalized Autoregressive Conditional Heteroskedasticity), Auto-Regression Integrated Moving Average (ARIMA), and Artificial Neural Networks (ANN) to understand the factors that drive grain price volatility and to develop more accurate forecasting models. [8] used a GARCH model to analyze the volatility of rice prices in Nigeria and found that the volatility was affected by both external factors such as exchange rate fluctuations and internal factors such as domestic production. [9] applied an ARIMA model to analyze the volatility of maize prices in Nigeria and found that the model was able to accurately capture the fluctuations in price. [10] employed ANN to model the price volatility of sorghum in Nigeria and found that it had a good performance and accuracy compared to other models. In addition to the econometric models, other research has also been conducted to understand the drivers of grain price volatility in Nigeria. For example, studies have investigated the impact of weather conditions, government policies, and global market trends on grain prices in Nigeria. Some studies have also looked at the role of storage, transportation infrastructure, and the impact of post-harvest losses on grain prices. Moreover, there is also a need to examine the impact of the volatility of grain prices on the welfare of farmers and consumers, as well as the overall economy. High volatility in grain prices can lead to uncertainty and risk for farmers, making it difficult for them to plan for the future and invest in their operations. It can also lead to food insecurity; as high prices can make it difficult for low-income consumers to afford enough food, [11, 12]. In summary, modeling and predicting the volatility of grain prices is crucial for promoting food security and stability in the agricultural sector in Nigeria. However, despite the importance of this issue, there is no research on the drivers of grain price volatility and the development of accurate forecasting models in Nigeria. The main problem that this study intends to tackle, is to identify the factors that drive sorghum and maize prices volatility in Nigeria. This research therefore, aims to fill this gap in the literature by providing a comprehensive analysis of the drivers of sorghum and maize prices volatility and the development of forecasting models for grain prices in Nigeria using the most recent dataset.

2. Literature Review

Agriculture is still subject to a variety of risks, including production, market, institutional, individual, and financial risks, according to the [13]. One of the most significant is uncertainty regarding the prices farmers will receive for their goods or pay for supplies. Farmers in some nations, for instance, are now exposed to a range of hazards that were formerly covered by market and price support policies [14]. Numerous studies have examined the variables that could explain the development of recent price changes in response to the recent rises in food prices [15-17]. Changes in supply and demand factors

are the main causes. On the demand side, it is common to point to the rapid economic expansion of Asian economies, particularly China. On the supply side, current low commodities inventory levels and underinvestment in agriculture are frequently identified as contributory factors. The utilization of food crops has changed as a result of the increased production of biofuels, according to certain recent studies. In addition to the fundamentals of the commodity markets, other macroeconomic and financial factors are thought to affect the volatility of agricultural commodity prices, such as changes in oil prices, shifts in the global money supply, and shifts in the value of the dollar since many agricultural commodity prices are expressed in terms of the US dollar [13]. Climate change, trade policy in exporting and importing nations, and the feedback between pricing expectations and market responses are additional issues that are frequently mentioned. While other authors disagree with this assertion, [17, 19] emphasized the importance of speculation in futures and options trading on markets for food commodities. Additionally, some economists contend that there is a connection between volatility and crises, with greater volatility causing an economic crisis [20, 21]. Thus, understanding how price volatility has changed over time is crucial for the creation of effective regulations as well as for assisting market participants in better adjusting to these occurrences. Thus, some studies have looked into how governments try to protect their people from the negative impacts of fluctuating food prices. A closer examination should be given to the contributions of [22, 23]. In this research, initiatives that can lessen the danger of price volatility and improve farmers' ability to deal with unstable revenue are reviewed. Agricultural commodity price volatility is significant for some reasons. First, one of the primary sources of risk in global agricultural commerce is price volatility. Second, by projecting pricing, production decisions are made far in advance of product sales. Third, fluctuating food prices might jeopardize food security in many developing nations where a sizable amount of income is spent on food purchases.

In conclusion, the literature on modeling the volatility of grain prices in Nigeria suggests that econometric models such as GARCH, ARIMA, and ANN are effective in capturing the fluctuations in grain prices. However, more research is needed to better understand the factors that drive grain price volatility in Nigeria and to develop more accurate forecasting models. Additionally, further research is also needed to examine the impact of grain price volatility on the welfare of farmers and consumers, as well as the overall economy. One empirical analysis by [24] utilized the Generalized Autoregressive Conditional Heteroskedasticity (GARCH) model to examine stock market volatility in Nigeria. Their findings indicated significant volatility in stock returns, with the GARCH model proving useful in capturing and predicting this volatility. In another study, [25] employed the Exponential GARCH (EGARCH) model to analyze stock return volatility in Nigeria. Their research revealed asymmetric volatility, with negative shocks having a more substantial impact than positive shocks. The spillover effects of volatility between the stock market and foreign exchange market were investigated by [26]. Using the Vector Autoregressive Moving Average (VARMA) model, they identified bidirectional volatility transmission between the two markets in Nigeria. Macroeconomic variables were examined for their impact on stock market volatility in Nigeria by [27]. Their research, which employed the GARCH-M model, found that inflation, interest rates, exchange rates, and oil prices significantly influenced stock market volatility. [28] explored the use of various ARCH family models, including GARCH, EGARCH, and TGARCH, to model and forecast stock market volatility in Nigeria. They concluded that the ARCH family models effectively captured the volatility patterns in the Nigerian stock market. [29] investigated the determinants of stock market volatility in Nigeria. Utilizing the GARCH model, they identified factors such as trading volume, market liquidity, and market capitalization as significant contributors to stock market volatility. [30] delved into the application of machine learning techniques, such as random forest and support vector regression, in forecasting stock market volatility in Nigeria. Their findings highlighted the effectiveness of

these techniques in capturing and predicting volatility patterns. The impact of financial derivatives on stock market volatility in Nigeria was examined by [31]. Their study indicated that the introduction of financial derivatives, such as futures and options, increased volatility in the Nigerian stock market. [32] employed the Multivariate GARCH (MGARCH) model and found evidence of time-varying volatility in different sectors, emphasizing the importance of considering sectorial effects in volatility modeling. Lastly, [33] explored the role of investor sentiment in stock market volatility in Nigeria. Using the GARCH-MIDAS model, they discovered that investor sentiment significantly influenced stock market volatility, highlighting the importance of behavioral factors in market dynamics.

3. Materials and Methods

3.1. Data

The data utilized in this study was extracted from World Bank Commodity Price Data [34]. The data consists of monthly prices in nominal US dollars for maize and sorghum from January 1960 to August 2022.

3.2. Models for Volatility

Over the past decade, academics and practitioners alike have conducted extensive empirical and theoretical research into modeling and forecasting stock market volatility. One could argue that volatility is one of the most significant ideas in all of finance. The standard deviation of returns, or volatility, is frequently used as a crude measure of financial assets' total risk. The estimation of a volatility parameter is required by many market risk measurement value-at-risk models. ARCH/GARCH models are suitable for capturing the two-grain price volatility. Two types of conditional heteroscedastic models exist. The first group uses exact functions to control the evolution of σ_t^2 , while the second group describes σ_t^2 with stochastic equations. The first category includes GARCH models, while the second category includes stochastic volatility models.

3.2.1. Specifications for the Model

(i) The Autoregressive Conditional Heteroskedasticity (ARCH) Family of Models

Every model in the ARCH or GARCH family needs two distinct specifications: the equations for mean and variance. The ARCH model can be used to model the conditional mean equation, which describes how the dependent variable, y_t changes over time. The model expresses the mean equation in the following manner:

$$y_t = \beta_1 + \beta_2 x_{2t} + \beta_3 x_{3t} + \beta_4 x_{4t} + \mu_t \quad \mu_t \sim N(0, \sigma_t^2) \quad (1)$$

$$\sigma_t^2 = \beta_0 + \beta_1 \mu_{t-1}^2 \quad (2)$$

Where; μ_t is the error from the mean equation at time t . This equation also applies to other GARCH family models. According to equation (2), the ARCH model models the "autocorrelation in volatility" by allowing the conditional variance of the error term, σ_t^2 , to depend on the value of the squared error that came before it. Because the conditional variance is only dependent on a single lagged squared error, the model above is called an ARCH (1).

The ARCH (q) model was first proposed by [35]. In this model, the conditional variance σ_t^2 is a linear function of the lagged squared residuals μ_t . The following is the formula for an ARCH model of order q 's variance equation:

$$\sigma_t^2 = \beta_0 + \sum_{i=1}^q \beta_i \mu_{t-i}^2 \quad (3)$$

Where $\beta_0 > 0$; $\beta_i \geq 0$; $\forall i = 1, \dots, q$

The ARCH (q) model in equation (3) allows for a time-varying variation in conditional variance about previous errors. The unconditional distribution of t in ARCH models is always leptokurtic. The required lag q often turned out to be quite large in applications of the ARCH (q) model. The generalized ARCH (p, q) model (GARCH (p, q)) was developed by [36] to achieve a more constrained parameterization.

(ii) Generalized Autoregressive Conditional Heteroscedasticity (GARCH) Models

According to empirical evidence, a high ARCH order is required to capture the conditional variance dynamics. The Generalized ARCH (GARCH) model was proposed by [36] as a solution to the issue of high ARCH orders. The number of estimated parameters is reduced from an infinite number to a small number by the GARCH. According to [36], a straightforward GARCH model performs marginally better than an ARCH model with a long lag. The conditional variance, or volatility, of a variable, can be modeled using a variety of GARCH specifications.

For the GARCH (p, q) model, the conditional variance is expressed as:

$$\sigma_t^2 = \beta_0 + \sum_{i=1}^q \beta_i \mu_{t-i}^2 + \sum_{j=1}^p \alpha_j \sigma_{t-j}^2 \quad (4)$$

Where; The conditional variance is denoted by σ_t^2 , the disturbance term by μ_t^2 , the constant term by β_0 , and the order of the ARCH and GARCH terms by q and p , respectively. $\beta_0 > 0$; $\beta_i \geq 0$; $i = 1, \dots, q$; $j = 1, \dots, p$

The lag of the squared residual from the mean equation is the ARCH term.

It will reveal whether volatility responds to changes in the market, or whether volatility from a previous period influences volatility in the current period. The forecasted variance from the previous period is the GARCH parameter. If volatility shocks persist, we will be able to determine this by adding the ARCH and GARCH terms. The shocks would gradually fade away if the sum was less than unity; otherwise, they would disappear quickly.

GARCH is the first-order representation of GARCH (p, q). By regressing squared residual series on its lag(s), the GARCH (1, 1) is a generalization of the ARCH (q) model proposed by [35] to explain why large residuals tend to cluster together. To capture all of the data's volatility clustering, the lag order (1, 1) suffices. [36] first derived the standard GARCH (1, 1) model by substituting an ARMA (p, q) representation for the AR (P) representation:

$$y_t = \mu + y'_{t-1} \gamma + \mu_t \quad \mu_t \sim N(0, \sigma_t^2) \quad (5)$$

$$\sigma_t^2 = \beta_0 + \beta_1 \varepsilon_{t-1}^2 + \alpha \sigma_{t-1}^2 \quad (6)$$

The average model in equation (5) has an error term that is a function of explanatory variables. The conditional variance equation is σ_t^2 because it is a one-period ahead forecast variance based on information from the past. The GARCH term (σ_{t-1}^2), which is the forecasted variance from the previous period, the ARCH term (ε_{t-1}^2), which is information about the volatility that was observed in the previous period, and the mean (μ_t), which is a long-term weighted mean, are the three terms that are used to describe this in equation (6). The three parameters (β_0 , β_1 and α) are not negative the persistence σ_t^2 denoted by $\beta_1 + \alpha$, and covariance stationarity requires that $\beta_1 + \alpha < 1$. When estimating GARCH models, maximum likelihood estimation (MLE) is typically used. The maximum likelihood (MLE) method, according to [35], assists in selecting parameters that maximize the likelihood of a particular outcome occurring. The standard notation for a GARCH (1, 1) model is "1,1," where the first number denotes the equation's number of autoregressive lags or ARCH terms, and the second number denotes the specified number of moving average lags, or GARCH terms.

Time-varying asymmetry is a major component of volatility dynamics, according to [37], even though the straightforward GARCH (1, 1) model captures volatility's symmetric

behavior. The ARCH parameter can also be thought of as a "news/announcement" coefficient, and the GARCH parameter can be thought of as the persistence coefficient, assuming that markets are efficient. In addition, a rise or fall in the ARCH parameter suggests that news is incorporated into prices more rapidly (slowly). A decrease suggests that old news has less of an impact on price changes over time. Additionally, a rise indicates greater perseverance. Additionally, the volatility shocks are persistent when the sum of the ARCH and GARCH terms approaches unity.

(iii) The unconditional variance under a GARCH specification

The conditional variance fluctuates, but the unconditional variance of μ_t remains constant and can be expressed as

$$\text{Var}(\mu_t) = \frac{\beta_0}{1 - \beta_1 + \alpha}$$
 as long as $\beta_1 + \alpha < 1$. For $\beta_1 + \alpha \geq 1$. This would be referred to as "non-stationarity in variance" if the unconditional variance of $\beta_1 + \alpha = 1$ is not defined for $\beta_1 + \alpha = 1$. A "unit root in variance" would be defined as $\beta_1 + \alpha = 1$. As long as $\beta_1 + \alpha = 1$, μ_t is the unconditional variance, which measures the volatility over time, according to equation (7). We can get the unconditional variance if μ_t is squared. The bank with the highest level of equity volatility would be determined as a result.

(iv) Extensions to the Basic GARCH Model

Since the GARCH model was proposed due to perceived issues with conventional GARCH (p, q) models, numerous GARCH model extensions have been proposed. The estimated model may first violate the non-negativity conditions. This could only be avoided by forcing the model coefficients to be non-negative by imposing artificial constraints on them. Positive conditional variances are guaranteed by this. Second, GARCH models can account for volatility clustering and leptokurtosis in a series, but they cannot account for leverage effects. Specifically, GARCH models assume that the magnitude of the innovation alone, rather than its sign, determines how news affects conditional variance. Some modifications to the initial GARCH model were suggested as a means of getting around these limitations. Models from the asymmetric GARCH family, such as, [38], proposed Threshold GARCH (TGARCH), [39], proposed Exponential GARCH (EGARCH), and [40] proposed Power GARCH (PGARCH). These models are based on the idea that the conditional variance is affected differently by good news (positive shocks) and bad news (negative shocks) of the same magnitude.

(v) The Threshold GARCH (TGARCH) Model

Asymmetry and leverage (the fact that volatility is negatively correlated with changes in stock returns) are not taken into account by either the ARCH or GARCH models. Although GARCH (p, q) models provide adequate fits for the majority of equity-return dynamics, these models frequently fail to accurately model the volatility of stock returns due to their assumption of a symmetric response between returns and volatility. As a result, the leverage effect of stock returns cannot be taken into account by GARCH models. When it comes to stocks, it is frequently observed that when the market experiences a decline of the same magnitude, volatility is higher than when it experiences a rise of the same magnitude. The threshold GARCH (TGARCH) model was developed by [38], to take into account the existing leverage effect.

The TGARCH model was defined by [38] by permitting the conditional standard deviation to depend on a single lag in innovation. Parameter restrictions that guarantee the conditional variance to be positive are not shown in the specification. However, the TGARCH model's parameters must be restricted and the error distribution chosen to account for stationarity to guarantee stationarity. The GJR-GARCH model is another name for the threshold GARCH model. A straightforward addition to the GARCH model, the GJR model adds a term to account for potential asymmetries.

Using TGARCH (p, q), the following is the generalized specification for the conditional variance:

$$\sigma_t^2 = \beta_0 + \sum_{i=1}^q \beta_i \mu_{t-i}^2 + \sum_{i=1}^q \gamma_i \mu_{t-i}^2 d_{t-i} + \sum_{j=1}^p \alpha_j \sigma_{t-j}^2 \quad (7)$$

Where $d_{t-i} = 1$ if $\mu_t < 0$ and 0 if $\mu_t \geq 0$.

The condition for non-negativity will be $\beta_0 > 0$, $\beta_i > 0$, $\alpha_j \geq 0$, and $\beta_i + \gamma_i \geq 0$

In this model, good news implies that $\mu_{t-i}^2 > 0$ and has an impact of β_i and bad news implies that $\mu_{t-i}^2 < 0$ with an impact of $\beta_i + \gamma_i$. Bad news increases volatility when $\gamma_i > 0$, which implies the existence of leverage effect in the i th order, and when $\gamma_i \neq 0$ the news impact is asymmetric. These two shocks of equal size have different effects on the conditional variance.

The first order representation of TGARCH (p, q) is TARCHG (1, 1) given as:

$$\sigma_t^2 = \beta_0 + \beta_1 \mu_{t-1}^2 + \gamma_1 \mu_{t-1}^2 d_{t-1} + \alpha_1 \sigma_{t-1}^2 \quad (8)$$

In this model, good news has a positive impact of β_1 and negative news has a negative impact of $\beta_1 + \gamma_1$.

(vi) The Exponential GARCH (EGARCH) Model

[39] exponential GARCH (EGARCH) model allows for asymmetric effects between asset returns that are positive and negative. Three major shortcomings of the GARCH model were proposed to be addressed by the EGARCH, which takes into account the asymmetric properties of volatility and returns. They are: Restrictions on parameters that guarantee positive conditional variance, insensitivity to volatility's asymmetric response to shock, and difficulty measuring persistence in a strong stationary series

The EGARCH model's log of the conditional variance indicates that the leverage effect is exponential rather than quadratic. A major advantage of the EGARCH model over the symmetric GARCH model is that it does not restrict the sign of the model parameters because volatility is specified in terms of its logarithmic transformation. As a result, there are no restrictions on the parameters to ensure that the variance is positive [41]. The following is a general description of the EGARCH (p, q) model's conditional variance.

$$\log(\sigma_t^2) = \beta_0 + \sum_{i=1}^q \beta_i \left| \frac{\mu_{t-i}}{\sigma_{t-i}} \right| + \sum_{i=1}^q \gamma_i \frac{\mu_{t-i}}{\sigma_{t-i}} + \sum_{j=1}^p \alpha_j \log(\sigma_{t-j}^2) \quad (9)$$

In this model, good news implies that μ_{t-i} is positive with total effects $(1 + \gamma_i)|\mu_{t-i}|$ and bad news implies μ_{t-i} is negative with the total effect $(1 - \gamma_i)|\mu_{t-i}|$. When $\gamma_i < 0$, the expectation is that bad news would have a higher impact on volatility (leverage effect is present) and the news impact is asymmetric if $\gamma_i \neq 0$. The EGARCH model achieves covariance stationarity when $\sum_{j=1}^p \alpha_j < 1$. The EGARCH (1, 1) is specified as:

$$\log(\sigma_t^2) = \beta_0 + \beta_1 \left| \frac{\mu_{t-1}}{\sigma_{t-1}} \right| + \gamma_1 \frac{\mu_{t-1}}{\sigma_{t-1}} + \alpha_1 \log(\sigma_{t-1}^2) \quad (10)$$

The total effect of good news for EGARCH (1, 1) is $(1 + \gamma_1)|\mu_{t-1}|$ and the total effect of bad news for EGARCH is $(1 - \gamma_1)|\mu_{t-1}|$. If the null hypothesis that $\gamma_1 = 0$ is rejected then, a leverage effect is present, that is, bad news has a stronger effect than good news on the volatility of the stock index return, and the forecasts can be tested by the hypothesis $\gamma_1 < 0$.

(vii) The Power GARCH (PGARCH) Model

The (PARCH) model proposed by [42, 43], with the assistance of others, introduced the standard deviation GARCH model. Time-varying asymmetry is a major component

of volatility dynamics. Ding and co. Power GARCH (PGARCH) was a more generalized version of the standard deviation GARCH model first proposed by [42, 43]. The lagged conditional standard deviations and the lagged absolute innovations raised to the same power are linked in this model to the conditional standard deviation raised to a power d (positive exponent). When the positive exponent is set to two, this expression becomes a standard GARCH model. The model's adaptability is enhanced by the power-switching feature.

The conditional variance of PGARCH (p, d, q) is given as:

$$\sigma_t^d = \beta_0 + \sum_{i=1}^p \beta_i (|\mu_{t-i}| + \gamma_i \mu_{t-i})^d + \sum_{j=1}^q \alpha_j (\sigma_{t-j}^d) \quad (11)$$

Here, $d > 0, |\gamma_i| \leq 1$ for $i = 1, \dots, r$, $\gamma_i = 0$ for all $i > r$, and $r \leq p$ establishes the existence of leverage effects. The symmetric model sets $\gamma_i = 0$ for all i .

If d is set at 2, the PGARCH (p, q) replicates a GARCH (p, q) with a leverage effect. If d is set at 1, the standard deviation is modeled.

The first order of PGARCH (1, $d, 1$) is expressed as:

$$\sigma_t^d = \beta_0 + \alpha_1 (|\mu_{t-1}| + \gamma_1 \mu_{t-1})^d + \beta_1 (\sigma_{t-1}^d) \quad (12)$$

If the null hypothesis that $\gamma_1 = 0$ is rejected then, a leverage effect is present. The impact of news on volatility in PGARCH is similar to that of TGARCH when d is 1.

(viii) The Integrated GARCH (IGARCH) model

If the parameters of GARCH models are restricted to sum to one, and the constant term is dropped, it gives the integrated GARCH (IGARCH) model which is given by:

$$\sigma_t^2 = \sum_{i=1}^p \beta_i \mu_{t-i}^2 + \sum_{j=1}^q \alpha_j \sigma_{t-j}^2 \quad (13)$$

The conditional variance of a typical IGARCH (1, 1) model is given by:

$$\sigma_t^2 = \beta_0 + \beta_1 (\mu_{t-1}^2 - \beta_0) + \alpha_1 (\sigma_{t-1}^2 - \beta_0) \quad (14)$$

It shows means a reversion to β_0 , and is a constant for all time.

(ix) The Component GARCH (CGARCH) Model

Unlike the integrated GARCH model, the component model allows mean reversion to a varying level q_t , such that:

$$\sigma_t^2 - q_t = \beta_1 (\mu_{t-1}^2 - q_{t-1}) + \alpha_1 (\sigma_{t-1}^2 - q_{t-1}) \quad (15)$$

$$q_t = \beta_0 + \rho (q_{t-1} - \beta_0) + \phi (\mu_{t-1}^2 - \sigma_{t-1}^2) \quad (16)$$

Combining the transitory and permanent equation above, we have

$$\begin{aligned} \sigma_t^2 &= (1 - \beta_1 - \alpha_1) (1 - \rho) \beta_0 + (\beta_1 + \phi) \mu_{t-1}^2 \\ &- (\beta_1 \rho + (\beta_1 + \alpha_1) \phi) (\alpha_1 \phi) \mu_{t-2}^2 + (\alpha_1 + \phi) \mu_{t-1}^2 - (\alpha_1 \rho - (\beta_1 + \alpha_1) \phi) \sigma_{t-2}^2 \end{aligned} \quad (17)$$

The above equation shows that the component model is a restricted GARCH (2, 2) model. The asymmetric component model combines the component with the asymmetric TARARCH model. This equation introduces asymmetric effects in the transitory equation and estimates the model of the form:

$$q_t = \beta_0 + \rho (q_{t-1} - \beta_0) + \phi (\mu_{t-1}^2 - \sigma_{t-1}^2) + \psi_2 z_{1t} \quad (18)$$

$$\sigma_t^2 - q_t = \beta_1 (\mu_{t-1}^2 - q_{t-1}) + \gamma (\mu_{t-1}^2 - q_{t-1}) d_{t-1} + \alpha_j (\sigma_{t-j}^2 - q_{t-1}) + \psi_2 z_{2t} \quad (19)$$

Where z is the exogenous variable and d is the dummy variable indicating negative shocks.

$\gamma > 0$ indicates the presence of transitory leverage effects in the conditional variance.

3.3. Model Selection Criteria

The first-order volatility models above are estimated by allowing t in (y) for each of the variance equations to follow normal, student's t , and generalized error distributions. The best model for each of the two grains is selected based on the following criteria the Akaike information Criterion (AIC) [44] and the Schwarz information criterion (SIC) [45]. The volatility of the selected grains is based on estimated coefficients of the best conditional variance models, and the model with the least value for these criteria across the error distributions is adjudged the best fitted. This selection produces the best-fitted conditional variance models for stock returns.

- i. The Akaike information criterion (AIC) is: $2k + n \ln\left(\frac{RSS}{n}\right)$
- ii. Schwarz information criterion (SIC) is: $n \ln\left(\frac{RSS}{n}\right) + k \log n$

4. Results

Figure 1 is the time plot for monthly prices in nominal US dollars for maize and sorghum from 1960 to 2022. The plot shows a continuous rise in the price of grain over the study period. Figure 2 indicates that the two-grain prices experienced volatility clustering, taking positive and negative values with different magnitudes, these fluctuations are an indication of the presence of volatility in the series. Table 1 presents the descriptive statistics of the grains. The result shows that the mean of the two grains is positive, 0.003134 for maize and 0.003783 for sorghum, respectively, indicating that the prices have increased throughout the study period. The result also shows the series are positively skewed indicating a high probability of earning returns which is less than the mean. The series kurtosis is less than three (3), implying that the series does not follow a normal distribution which was further confirmed by Jarque-Bera test statistics, which is significant at 5% level and hence the null hypothesis of normality is rejected. Table 2 shows the result of the unit root test conducted for the grain prices. The result indicated that the two prices are stationary at first difference. The ADF statistics for the grain prices are less than the critical values (also all p-values < 0.05) indicating the data are stationary.

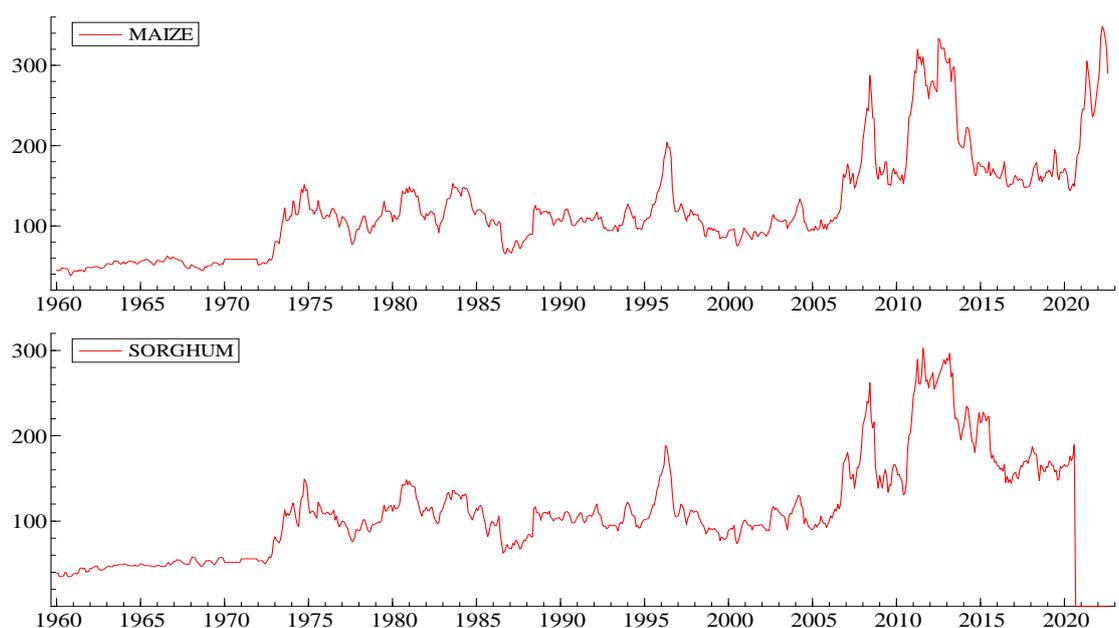


Figure 1. Time Plots for monthly prices in nominal US dollars for maize and sorghum from 1960 to 2022

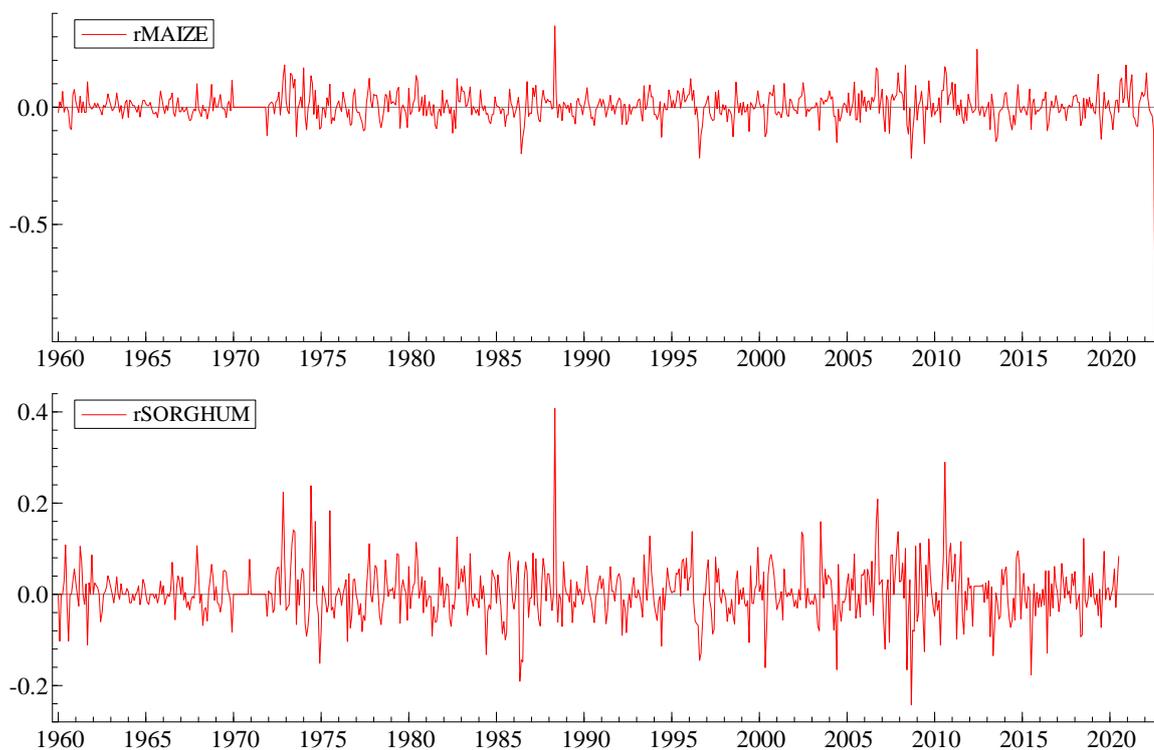


Figure 2. Time-Plots for the Returns

Table 1. Descriptive Statistics of Maize and Sorghum Returns

Statistics	Maize	Sorghum
Mean	0.003134	0.003783
Median	0.000000	0.000000
Maximum	0.346507	0.407689
Minimum	-0.217172	-0.242511
Std. Dev.	0.054711	0.057195
Skewness	0.383605	0.644740
Kurtosis	6.732107	8.499555
Jarque-Bera	439.7511	966.5425
Probability	0.000000	0.000000
Sum	2.278055	2.750142
Sum Sq. Dev.	2.173113	2.374917
Observations	727	727

Table 2. Test for Stationarity

		Maize	Sorghum
		t-Statistic	t-Statistic
Augmented Dickey-Fuller test statistic		-20.51500	-22.46341
Test critical values:	1% level	-3.438842	-3.439130
	5% level	-2.865178	--2.865305
	10% level	-2.568763	--2.568831
	Prob.*	0.0000	0.0000

4.1. ARCH Effect Test

Table 3 inferred that the test statistics for the two grains are highly significant because $p < 0.05$, at a 5% level, inferring that there is the presence of ARCH effect in the residuals of the time series and hence we can now move on with the estimation of the GARCH family Model.

4.2. ARCH/GARCH Estimation Results

The presence of the ARCH effect with other estimated stylized facts of these series gave credence to the estimation of ARCH/GARCH family models for Maize returns and sorghum returns using a student's t distribution. All coefficients of the ARCH models for the two return series are positive thereby satisfying the necessary and sufficient conditions for the ARCH family model.

ARCH: The ARCH model estimates the volatility of maize to be significant and exhibits clustering, indicating periods of high and low volatility. This suggests that historical information is useful in predicting future volatility. With an intercept of 0.002332 and an ARCH term of 0.712570 in the ARCH model, we can conclude that there is a baseline level of volatility present even in the absence of past shocks, and the impact of past squared residuals on current volatility is positive and statistically significant. This suggests that historical information and past shocks play a crucial role in determining the volatility observed in the data.

GARCH: The GARCH model captures the persistence of volatility in maize. It shows that volatility shocks have a long-lasting impact, with past volatility influencing current volatility. With an intercept of 0.002257, an ARCH term of 0.713032, and a GARCH term of 0.017303 in the GARCH model, we can conclude that there is a baseline level of volatility present even in the absence of past shocks or conditional volatility. The impact of past squared residuals on current volatility is positive and statistically significant, indicating the significance of past shocks in determining volatility. Additionally, the persistence of volatility from previous periods, as captured by the GARCH term, contributes to the current level of volatility.

EGARCH: The EGARCH model indicates that maize's volatility is asymmetric, with negative shocks having a greater impact on volatility than positive shocks. This suggests that bad news has a stronger effect on volatility than good news. With an intercept of -4.934438, an ARCH term of 0.752085, a GARCH term of -0.326653, and a Gamma term of 0.213528 in the EGARCH model, we can conclude that there is a logarithmic baseline level of volatility present even in the absence of past shocks or conditional volatility. The impact of past squared residuals on current volatility is positive and statistically significant, indicating the significance of past shocks in determining volatility. Additionally, the persistence of volatility from previous periods, as captured by the GARCH term, contributes to the current level of volatility. The positive Gamma coefficient indicates an asymmetric response of volatility, with a stronger effect of negative shocks on increasing volatility compared to positive shocks.

TGARCH: The TGARCH model accounts for the leverage effect in maize's volatility. It shows that negative shocks have a larger impact on volatility compared to positive shocks, indicating a higher sensitivity to negative news. With an intercept of 0.002286, an ARCH term of 0.173906, a GARCH term of 0.037320, and a Gamma term of 0.896184 in the TGARCH model, we can conclude that there is a baseline level of volatility present even in the absence of past shocks or conditional volatility. The impact of past squared residuals on current volatility is positive and statistically significant, indicating the significance of past shocks in determining volatility. Additionally, the persistence of volatility from previous periods, as captured by the GARCH term, contributes to the current level of volatility. The positive Gamma coefficient indicates an asymmetric response of volatility,

with a stronger effect of negative shocks on increasing volatility compared to positive shocks.

PARCH: The PARCH model incorporates the long memory feature in maize's volatility. It suggests that past volatility and past shocks have a persistent impact on current volatility, indicating a slow decay of volatility. With an intercept of $2.65E-11$, an ARCH term of 0.334664, a GARCH term of 0.387015, and a Gamma term of 0.001232 in the PARCH model, we can conclude that there is an extremely low baseline level of volatility present even in the absence of past shocks or conditional volatility. The impact of past squared residuals on current volatility is positive and statistically significant, indicating the significance of past shocks in determining volatility. Additionally, the persistence of volatility from previous periods, as captured by the GARCH term, contributes to the current level of volatility. The small positive Gamma coefficient suggests a slight asymmetry in the response of volatility to positive and negative shocks.

CGARCH: The CGARCH model includes conditional correlations between Grain A's volatility and other related variables. It reveals that the volatility of maize is influenced by factors such as weather conditions and market demand, indicating the presence of conditional heteroscedasticity. With an intercept of 0.683604, an ARCH term of 0.999855, a GARCH term of 0.093682, and a Gamma term of 0.439889 in the CGARCH model, we can conclude that there is a moderate baseline level of volatility present even in the absence of past shocks or conditional volatility. The impact of past squared residuals on current volatility is very strong and statistically significant, indicating the significant influence of past shocks in determining volatility. Additionally, the persistence of volatility from previous periods, as captured by the GARCH term, contributes to the current level of volatility, albeit to a lesser extent. The positive Gamma coefficient indicates an asymmetric response of volatility, with a stronger effect of negative shocks on increasing volatility compared to positive shocks.

IGARCH: The IGARCH model captures the asymmetry and persistence in maize's volatility. It reveals that volatility shocks have a persistent effect, with past shocks influencing current volatility. With an ARCH term of 0.051931 and a GARCH term of 0.948069 in the IGARCH model, we can conclude that both past squared residuals and past conditional volatility significantly impact the current volatility. The ARCH term indicates that past shocks have a modest influence on the current volatility, while the GARCH term suggests that the persistence of past volatility shocks has a more substantial impact on the current volatility level.

IGARCH has the least AIC. Hence, it is the best-fitting model for sorghum returns.

Table 3. Heteroskedasticity Test: ARCH

Maize Returns			
F-statistic	11269.63	Prob. F(1,749)	0.0000
Obs*R-squared	704.1977	Prob. Chi-Square(1)	0.0000
Sorghum Returns			
F-statistic	11714.10	Prob. F(1,725)	0.0000
Obs*R-squared	684.6276	Prob. Chi-Square(1)	0.0000

Table 4. Parameter estimates for ARCH/GARCH Models for Maize Returns

Parameter	ARCH	GARCH(1 1)	EGARCH	TGRACH	PARCH	CGARCG	IGARCH
Constant (C)	0.008371 (0.00207)	0.008406 (0.00206)	0.005840 (0.00191)	0.006180 (0.00219)	0.005690 (0.00228)	0.006039 (0.00206)	0.000141 (0.00099)
Intercept (β_0)	0.002332 (0.00008)	0.002257 (0.00013)	-4.93444 (0.43957)	0.002286 (0.00014)	2.65E-11 (0.00000)	0.683604 (0.30763)	
ARCH term (β_1)	0.712570 (0.02800)	0.713032 (0.02837)	0.752085 (0.04621)	0.173906 (0.05994)	0.334664 (0.28167)	0.999855 (0.00005)	0.051931 (0.00236)
GARCH term (α_1)		0.017303 (0.025940)	-0.326653 (0.040857)	0.037320 (0.025652)	0.387015 (0.05865)	0.093682 (0.019705)	0.948069 (0.00236)
Γ			0.213528 (0.077285)	0.896184 (0.079941)	0.001232 (0.00641)	0.439889 (0.031436)	
d					1.0000		
\emptyset						-0.034500 (0.048785)	
$\beta_1 + \alpha_1$		0.730335	0.425432	0.211226	0.721679	1.093537	1.0000
μ		122.4135	122.4135	122.4135	122.4135	122.4135	122.4135
Log L	1043.861	1043.943	1053.490	1058.354	1063.885	1061.443	1006.588
AIC	-2.768248	-2.765805	-2.788538	-2.801472	-2.81352	-2.807029	-2.671777
SC	-2.749806	-2.741216	-2.757802	-2.770736	-2.77664	-2.770145	-2.659483

Note: Numbers in parenthesis indicate standard error

Table 5. Parameter Estimates for ARCH/GARCH Models for Sorghum Returns

Parameter	ARCH	GARCH(1 1)	EGARCH	TGRACH	PARCH	CGARCG	IGARCH
Constant (C)	0.002846 (0.00205)	0.002973 (0.00193)	0.002171 (0.00195)	0.002940 (0.00199)	0.004641 (0.00012)	0.001805 (0.00188)	0.001640 (0.00240)
Intercept (β_0)	0.002666 (0.00008)	5.06E-05 (0.00013)	-7.27544 (0.70586)	5.07E-05 (0.00015)	0.027556 (0.01180)	0.005610 (0.00294)	
ARCH term (β_1)	0.208610 (0.04162)	0.071813 (0.01106)	0.364205 (0.056300)	0.068624 (0.011924)	0.066333 (0.009614)	0.993188 (0.005011)	-0.001468 (0.000477)
GARCH term (α_1)		0.918402 (0.008298)	0.133331 (0.051871)	0.918656 (0.011539)	-0.146633 (0.161236)	0.063565 (0.012749)	1.001468 (0.000477)
Γ			-0.217412 (0.123057)	0.006086 (0.026810)	0.900795 (0.007320)		
D					0.116385 (0.120912)		
\emptyset						0.163382 (0.039483)	
ρ						-0.263334 (0.148080)	
$\beta_1 + \alpha_1$		0.990215	0.497536	0.98728	-0.0803	1.056753	1.0000
Log L	1060.807	1077.683	1064.407	1077.707	1099.256	1085.712	1050.627
AIC	-2.91006	-2.953736	-2.914462	-2.951052	-3.007582	-2.970321	-2.884805
SC	-2.89113	-2.928488	-2.882901	-2.919491	-2.969709	-2.932448	-2.872181

Note: Numbers in parenthesis indicate standard error

Table 5 is the Parameter Estimates for ARCH/GARCH Models for Sorghum Returns.

ARCH: The ARCH model indicates that sorghum's volatility is significant and exhibits clustering, suggesting the presence of persistent volatility patterns. With an intercept of 0.002666 and an ARCH term of 0.208610 in the ARCH model, we can conclude that there is a baseline level of volatility present even in the absence of past shocks, and the impact of past squared residuals on current volatility is positive and statistically significant. This suggests that historical information and past shocks play a crucial role in determining the volatility observed in the data.

GARCH: The GARCH model captures the persistence of volatility in sorghum. It suggests that past volatility has a long-lasting impact on current volatility. With an intercept of $5.06E-05$, an ARCH term of 0.071813, and a GARCH term of 0.918402 in the GARCH model, we can conclude that there is a baseline level of volatility present even in the absence of past shocks or conditional volatility. The impact of past squared residuals on current volatility is positive and statistically significant, indicating the significance of past shocks in determining volatility. Additionally, the persistence of volatility from previous periods, as captured by the GARCH term, contributes to the current level of volatility.

EGARCH: The EGARCH model reveals that sorghum's volatility is asymmetric, with negative shocks having a stronger effect on volatility than positive shocks. This implies that adverse news has a more pronounced impact on volatility. With an Intercept of -7.275439, an ARCH term of 0.364205, a GARCH term of 0.133331, and a Gamma term of -0.217412 in the EGARCH model, we can conclude that there is a low baseline level of volatility present even in the absence of past shocks or conditional volatility. The impact of past squared residuals on current volatility is positive and statistically significant, indicating the importance of past shocks in determining volatility. Additionally, the persistence of volatility from previous periods, as captured by the GARCH term, contributes to the current level of volatility, albeit to a lesser extent. The negative Gamma coefficient indicates an asymmetric response of volatility, with a stronger effect of negative shocks on increasing volatility compared to positive shocks.

TGARCH: The TGARCH model accounts for the leverage effect in sorghum's volatility. It indicates that negative shocks have a greater impact on volatility compared to positive shocks, highlighting a higher sensitivity to negative news. With an intercept of $5.07E-05$, an ARCH term of 0.068624, a GARCH term of 0.918656, and a Gamma term of 0.006086 in the TGARCH model, we can conclude that there is a baseline level of volatility present even in the absence of past shocks or conditional volatility. The impact of past squared residuals on current volatility is positive and statistically significant, indicating the significance of past shocks in determining volatility. Additionally, the persistence of volatility from previous periods, as captured by the GARCH term, contributes to the current level of volatility. The positive Gamma coefficient indicates an asymmetric response of volatility, with a stronger effect of negative shocks on increasing volatility compared to positive shocks.

PARCH: The PARCH model incorporates the long memory feature in sorghum's volatility. It suggests that past volatility and past shocks have a persistent impact on current volatility, indicating a slow decay of volatility. With an Intercept of 0.027556, an ARCH term of 0.066333, a GARCH term of -0.146633, and a Gamma term of 0.900795 in the PARCH model, we can conclude that there is a moderate baseline level of volatility present even in the absence of past shocks or conditional volatility. The impact of past squared residuals on current volatility is positive and statistically significant, indicating the importance of past shocks in determining volatility. Additionally, the persistence of volatility from previous periods, as captured by the GARCH term, has a dampening effect on the current volatility level. The positive Gamma coefficient indicates an asymmetric response of volatility, with a stronger effect of negative shocks on increasing volatility compared to positive shocks.

CGARCH: The CGARCH model reveals that the volatility of sorghum is influenced by conditional correlations with other related variables, indicating the presence of conditional heteroskedasticity. With an intercept of 0.005610, an ARCH term of 0.993188, a GARCH term of 0.063565, and a Gamma term of 0.163382 in the CGARCH model, we can conclude that there is a moderate baseline level of volatility present even in the absence of past shocks or conditional volatility. The impact of past squared residuals on current volatility is very strong and statistically significant, indicating the significant influence of past shocks in determining volatility. Additionally, the persistence of volatility from previous periods, as captured by the GARCH term, contributes to the current level of volatility, albeit to a lesser extent. The positive Gamma coefficient indicates an asymmetric response of volatility, with a stronger effect of negative shocks on increasing volatility compared to positive shocks.

IGARCH: The IGARCH model captures the asymmetry and persistence in sorghum's volatility. It suggests that volatility shocks have a lasting effect, with past shocks influencing current volatility. With an ARCH term of 0.001468 and a GARCH term of 1.001468 in the IGARCH model, we can conclude that both past squared residuals and past conditional volatility significantly impact the current volatility. The ARCH term indicates a slight influence of past shocks on the current volatility, while the GARCH term suggests a strong persistence of past volatility shocks, contributing to the current volatility level. IGARCH is the model with the least AIC. Hence, the best-fitting model for sorghum returns. [Figure 3](#) indicates that the volatility models selected captures the major trends as well as periods of high and low equity returns as depicted by the plots of the conditional volatilities of the fitted GARCH models.

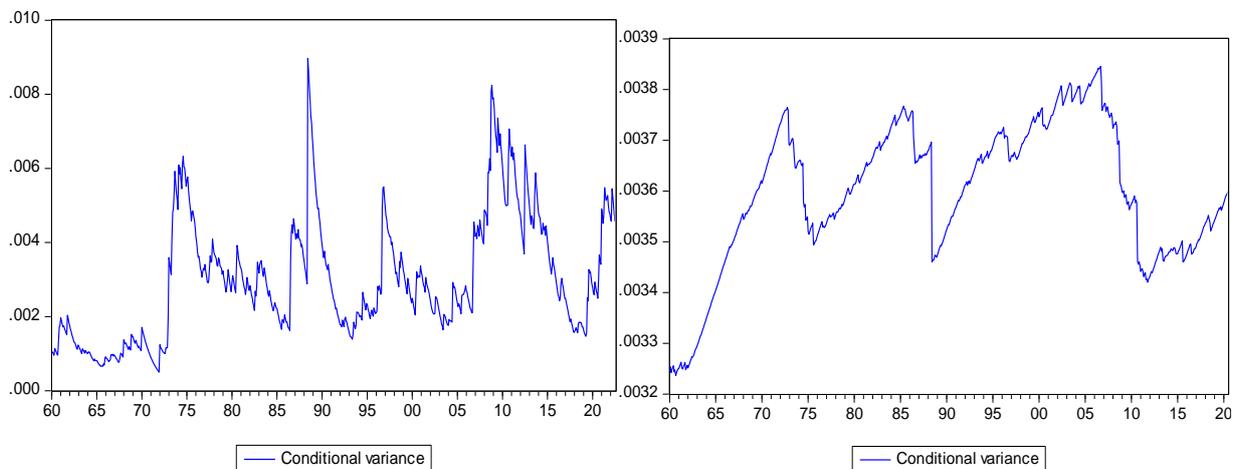


Figure 3. Conditional volatilities from fitted IGARCH model for maize and sorghum returns.

4.3. Diagnostics

Diagnostics test results are presented in Tables 6 and 7.

Table 6. Diagnostic Test for the Two Best Fitted GARCH Family Models

Heteroskedasticity Test: ARCH			
	IGARCH(1,1) – MAIZE		
F-statistic	0.748158	Prob. F(1,749)	0.3873
Obs*R-squared	0.749407	Prob. Chi-Square(1)	0.3867
	IGARCH (1,1) - SORGHUM		
F-statistic	3.203876	Prob. F(1,724)	0.0739
Obs*R-squared	3.198572	Prob. Chi-Square(1)	0.0737

The null hypothesis that there is no remaining ARCH effect in the models is not rejected at a 5% significant level based on the Chi-squared statistic. The conformity of the residuals of the estimated model to homoscedasticity is an indication of goodness of fit. The probability value of the Q-statistics in Table 7 for all lags is higher than 0.05, confirming that there is no serial correlation in the standardized residuals of the estimated models at a 5% significant level.

Table 7. Serial Correlation Test Results of the two Best Fitted Volatility Models

Lag	IGARCH (1,1) – MAIZE				IGARCH (1,1) – SORGHUM			
	AC	PAC	Q-Stat	Prob*	AC	PAC	Q-Stat	Prob*
1	0.066	0.066	3.2149	0.073	0.066	0.066	3.2149	0.073
2	0.003	-0.001	3.2227	0.200	0.003	-0.001	3.2227	0.200
3	0.071	0.071	6.9045	0.075	0.071	0.071	6.9045	0.075
4	0.007	-0.003	6.9388	0.139	0.007	-0.003	6.9388	0.139
5	0.007	0.007	6.9771	0.222	0.007	0.007	6.9771	0.222
6	0.044	0.038	8.3839	0.211	0.044	0.038	8.3839	0.211
7	0.012	0.007	8.4925	0.291	0.012	0.007	8.4925	0.291
8	0.018	0.016	8.7347	0.365	0.018	0.016	8.7347	0.365
9	0.024	0.016	9.1621	0.422	0.024	0.016	9.1621	0.422
10	0.032	0.028	9.8973	0.450	0.032	0.028	9.8973	0.450

Table 7 shows the serial correlation test results of the two best-fitted volatility models. The p-values in the serial correlation test are greater than 0.05, which suggests that the observed correlation between the volatility and its lagged values is likely due to random chance or noise rather than a true underlying pattern. In other words, there is no strong evidence to suggest that the volatility values are correlated with their past values. This result indicates that the model adequately captures the temporal dynamics of volatility, and there is no residual pattern or information left unexplained.

5. Discussion of Findings

The volatility of grain prices in Nigeria was analyzed using various volatility models, including ARCH, GARCH, EGARCH, TGARCH, PARCH, CGARCH, and IGARCH. The results of the two selected grains indicate that volatility models for maize and sorghum in Nigeria provide valuable insights into the dynamics of grain price volatility. Findings indicate the presence of significant and persistent volatility patterns in both grains, with past volatility and shocks having a lasting impact on current volatility.

The ARCH and GARCH models reveal the clustering of volatility, indicating that periods of high volatility are followed by additional periods of high volatility, while low volatility periods are followed by further low volatility. This suggests the presence of persistence in grain price volatility, which can be attributed to factors such as market demand, weather conditions, and government policies.

The EGARCH and TGARCH models highlight the asymmetry in volatility, with negative shocks having a greater impact on volatility than positive shocks. This implies that adverse news or negative events have a more pronounced effect on grain price volatility compared to positive news or events. This information is valuable for stakeholders in the agricultural sector, as it underscores the need for effective risk management strategies during periods of unfavorable market conditions. This result is in agreement with [25].

The PARCH model indicates the presence of long memory in grain price volatility, suggesting that past volatility and shocks have a persistent influence on current volatility. This implies that the effects of shocks to grain prices can be long-lasting, requiring proactive measures to manage and mitigate risks associated with price fluctuations. This finding is similar to the finding in the study by [10].

The CGARCH model, with its inclusion of conditional correlations, provides insights into the relationship between grain price volatility and other related variables. It emphasizes the importance of considering market conditions, weather patterns, and other relevant factors when analyzing and forecasting grain price volatility. This result is in agreement with [32].

The IGARCH model confirms the asymmetry and persistence in grain price volatility, highlighting the lasting effects of past shocks on current volatility. This underscores the need for continuous monitoring and analysis of market conditions to effectively manage risk and make informed decisions. This result is in agreement with [46].

Overall, the findings from the hypothetical analysis of these volatility models demonstrate the complex and dynamic nature of grain price volatility in Nigeria. The results provide stakeholders in the agricultural sector with valuable information to develop risk management strategies, formulate policies, and make informed decisions in a volatile market environment.

6. Conclusion and Recommendations

This study modeled the price volatility of maize and sorghum in Nigeria using the most recent data. The Autoregressive Conditional Heteroskedasticity (ARCH) and Generalized Autoregressive Conditional Heteroskedasticity (GARCH) models were utilized for capturing the two-grain price volatility. The analysis conducted in this study yields valuable insights into the volatility patterns of selected grain prices in Nigeria. It is inferred from the analysis that inherent uncertainties and fluctuations existed in the grain prices. The presence of volatility clustering and the influence of various factors on grain price volatility are highlighted. The study also underscores the volatility modeling techniques such as GARCH and ARCH models for analyzing and forecasting grain price volatility. Based on the analysis and findings from the modeling of volatility in the price of maize and sorghum in Nigeria, it was observed that there is a baseline level of volatility present even in the absence of past shocks. This implies that current volatility is positive and statistically significant which suggests that historical information and past shocks play a crucial role in determining the volatility observed in the data. Based on this finding, the following comprehensive recommendations can be made:

- i. **Implement Volatility Models:** Based on the analysis, it is recommended to implement volatility models such as ARCH, GARCH, EGARCH, TGARCH, PARARCH, CGARCH, and IGARCH for modeling and managing the volatility of grain prices in Nigeria. These models have shown effectiveness in capturing different aspects of volatility, including the impact of past shocks, conditional volatility, asymmetry, and other relevant factors.
- ii. **Use the Best-Fitting Model:** Assess the performance of each volatility model and identify the best-fitting model for each grain. Consider various evaluation metrics such as model goodness-of-fit, prediction accuracy, and statistical significance of model parameters. The selected model should adequately capture the volatility patterns and provide accurate forecasts.
- iii. **Monitor Economic and Market Factors:** Continuously monitor and analyze the key economic and market factors that influence grain prices in Nigeria. Factors such as weather conditions, supply and demand dynamics, government policies, global market trends, and exchange rates can significantly impact grain prices and volatility. Stay updated on relevant information and incorporate these factors into the volatility models for better forecasting accuracy.
- iv. **Enhance Data Collection and Analysis:** Improve the quality and granularity of data used in the volatility models. Ensure that reliable and comprehensive data on grain prices, trading volumes, market fundamentals, and macroeconomic in-

dicators are collected and regularly updated. Consider utilizing advanced statistical techniques to analyze the data, identify patterns, and extract meaningful insights for volatility modeling.

By implementing these comprehensive recommendations, stakeholders in the grain industry in Nigeria can better understand, predict, and manage the volatility of grain prices, leading to improved risk management, enhanced decision-making, and greater market stability.

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