Article

Modelling Population Growth Prognosis

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Abstract: Logistic growth model and its variants have been adjudged to be the most appropriate model for forecasting human population. However, in this article, we estimated the carrying capacity of Abuja using the logistic model. Then, we presented the parameters used to ascertain that the logistic model has the best fit in modelling population growth of Abuja over time. Meanwhile, a population growth sensitivity analysis is presented for the year 1962 to 2200.The result shows that by the year 2050, Abuja population growth rate will be out of control, if nothing substantial is implemented. Similarly, from the year 2150, the results show that stability will return again. Furthermore, the result of the error analysis conducted on the logistic model shows that Abuja has a growing population and that logistic growth model with MAPE and RMSE values of 0.98% and 7,817.07 respectively is the most accurate. The study concludes that logistic growth model with R−squared value of 0.776 has the best fit for population growth projection of Abuja. With approximate growth rate at 9.3% per annum, the projected population of Abuja will hit 30,220,701 million by the year 2039 all things being equal. Therefore, we recommend that the government should invest in massive agricultural reforms to accommodate the growing population, expand Abuja by developing its suburbs, and engage in massive reorientation of the populace on the dangers of uncontrolled births and the education of the girl child.

Keywords: Census; Population Growth; Carrying Capacity; Population Size

1. Introduction

Nigeria is one of the fastest growing countries in the world, with an estimated population of over 180 million and an annual population growth rate of 2.9%. Nigeria is the most populous black nation in Sub-Sahara Africa and the tenth most populous in the world according to [1-3]. This sounds interesting, but the question is, do the government have appropriate data and policy direction for effective monitoring and control of the population growth rate in such a way that the growing population maintains stability? No doubt it is the duty of government to monitor and manage population and demography related issues. In 1988, the armed forces ruling council headed by General Ibrahim Babangida, approved a policy document titled "Nigeria Policy on Population for Unity, Progress and Self-reliance" [4]. This policy was a proof of government seriousness and concern for family planning as part of the overall socio-economic development of the country. In 2004, another policy of government was introduced by President Olusegun Obasanjo Administration. This document was called" Nigeria Policy on Population for Sustainable Development" [5]. The policy recognizes that population factors, social and economic development, and environmental issues are irrevocably inter-related and are therefore critical to the achievement of sustainable development in Nigeria. Since then, several administrations have had one birth control measure or the other, but all of these policies have failed to categorically state the result of the population sensitivity analysis

which informed such hastily decision seven though it has good intentions. Presently, there is no population sensitivity analysis data across the nation and in particular Abuja. In an attempt to fill the gap, [6] used a mathematical approach and concluded that by 2040,the population growth rate of Nigeria will be out of control stating that everything should be done by the government both in policy and enforcement to ensure that the population growth is been checked early enough and controlled. Conversely, in this article, a logistic population growth model that incorporates a theoretical carrying capacity is investigated. The stability of the growing population is analyzed using the population growth sensitivity analysis. The expected time of instability is presented and recommendations given.

Study Area: Abuja also known as center of unity is the capital of Nigeria is one of the fastest growing cities in the world with a projected population of over 3 million in 2023 with an annual growth rate of 9.3% [7]. Geographically, Abuja is located on coordinates 9^0 4′N and 7^0 29′E covering an area of 570 sq miles and a population density of Abuja is located north of the confluence of River Niger and River Benue. It is bordered by the states of Niger to the West and North, Kaduna to the northeast, Nasarawa to the east and south, and Kogi to the southwest. The indigenous tribe in Abuja includes; Koro, Gbari, Gade, Nupe, Gwandara, Dibo, Bassa, Ganagana, and Ebira, and other settler groups like the Hausa, Igbo and Yoruba [8]. Abuja is currently made up of six local government areas, namely; Abaji, Abuja Municipal, Bwari, Gwagwalada, Kuje and Kwali. Abuja was the centre of extensive trading in the days preceding British colonization. A wide range of products were involved in the intense trading activity, attesting to the degree of diversification in the economy which, though largely agrarian, had a vibrant manufacturing sector whereby sheanut oil, honey, benniseed, locust bean cakes, ginger, peppers, kola nuts and rice were complemented by iron products, textile, mats leather items and pottery. Abuja was carved out from Niger, Plateau and Kwara states in 1976.

2. Materials and Methods

Exponential Model: The exponential model is represented by the differential equation

$$\frac{dN}{dt} = rN$$

with initial condition $N(0) = N_0$. Integrating and multiplying both sides by $\frac{dt}{N}$ gives

$$\int_0^t \frac{1}{N} = \int_0^t rsds$$

Therefore,

$$N(t) = N_0 e^{rt}$$

where the number of population at time t is $N(t)$, $N(0) = N_0$ is the initial population size, r is the population growth rate and $t = time$. Malthus proposed the exponential model. It is assumed that there are unlimited resources to sustain growth and there is no competition within the population.

Logistic Growth Model: The logistic growth model is represented by the differential equation

$$\frac{dN}{dt} = rN\left(1 - \frac{N}{K}\right)$$

where r is the population growth rate and K is the carrying capacity for $N(t)$ as $t \to \infty$.

In the logistic model, a population grows until it attains a maximum capacity [6]. Verhulst proposed the logistic model and it does not assume unlimited resources. Instead,

it assumes there is a carrying capacity K for the population. If the population is above K, then the population will decrease, but if below, then it will increase. The solution to the logistic equation can be obtained by applying the separation of variables method. From the equation above, we have:

$$N(t) = \frac{K}{1 + Ae^{-rt}}$$

Where

$$A = \frac{K - N_0}{N_0}$$

Then, the equation above is the classical logistic growth model. Furthermore, the logistic model can be modified and written as

$$N(t) = \frac{K}{1 + Ak^t}$$

Where $k = e^{-r}$.

Theorem 2.1. The least upper bound of the logistic model

$$N(t) = \frac{K}{1 + Ak^t} = \frac{K}{2}.$$

Proof. From the definition of the least upper bound, we have

$$2K > (K - 2\epsilon)(1 + Akt)$$

Hence,

$$k^t < \frac{K + 2\epsilon}{A(K - 2\epsilon)}$$

The application of a little algebra gives us

$$t < ln\left|\frac{K + 2\epsilon}{A(K - 2\epsilon)}\right| - ln\,k.$$

Population Growth Sensitivity: To calculate the growth sensitivity of the predicted population, we differentiate the sensitivity model using the quotient rule. Hence

$$\frac{dN(t)}{dt} = \frac{-KAk^t r}{[1 + Ak^t]^2}.$$

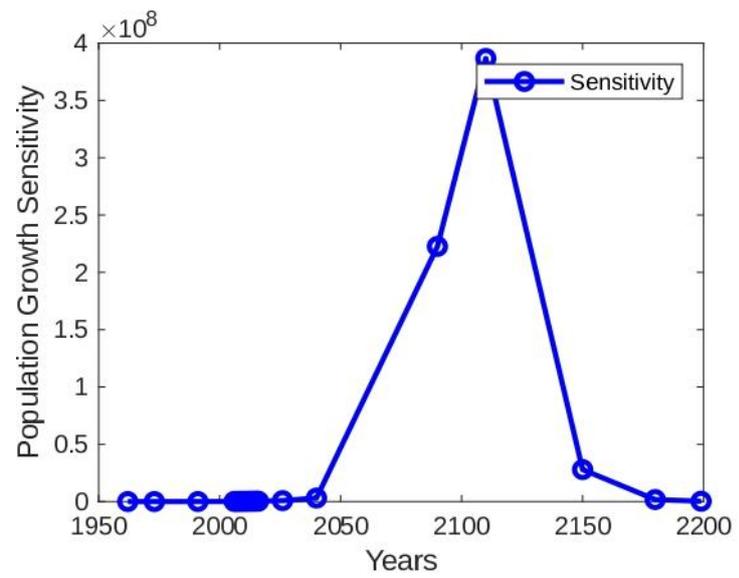Then the equation above is known as population growth sensitivity model.

3. Results
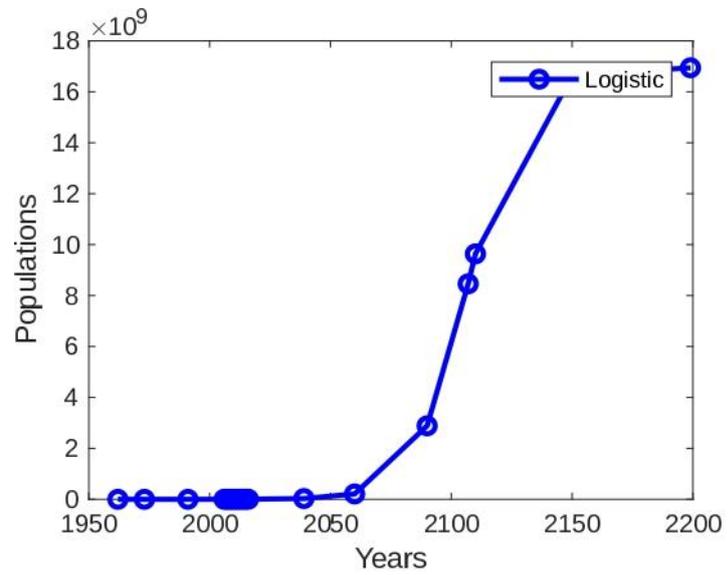
Population Data Visualization: The population data visualization for Abuja from 1962–2019 is plotted as demonstrated in Figures 1, 2, 3, 4 and 5 using Matlab R2023a;
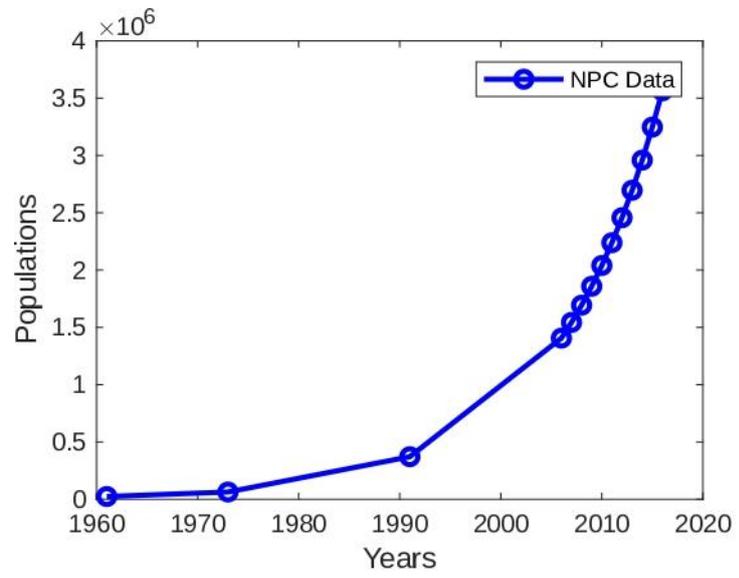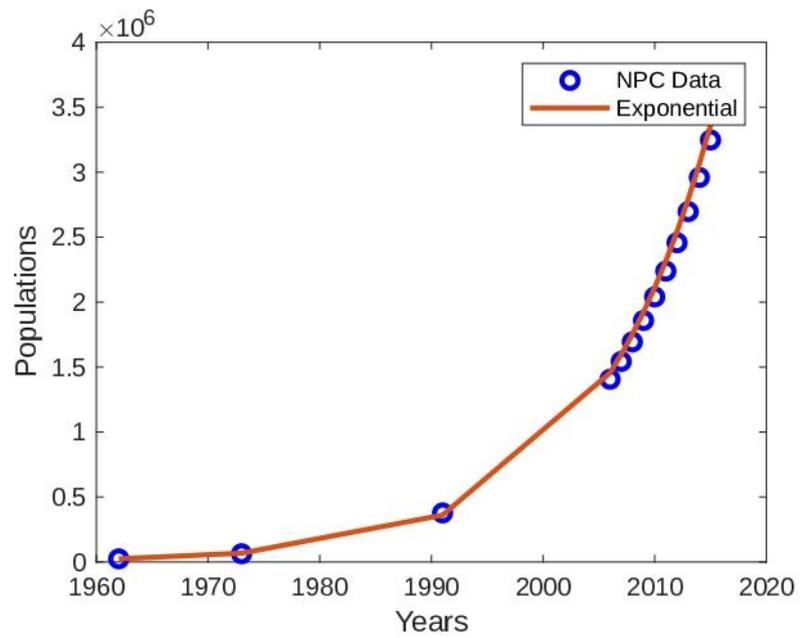
Figure 1. Population raw data simulation for Abuja

Figure 2. Growth Sensitivity simulation for Abuja

Figure 3. Logistic Model simulation for Abuja for 1962 to 2200

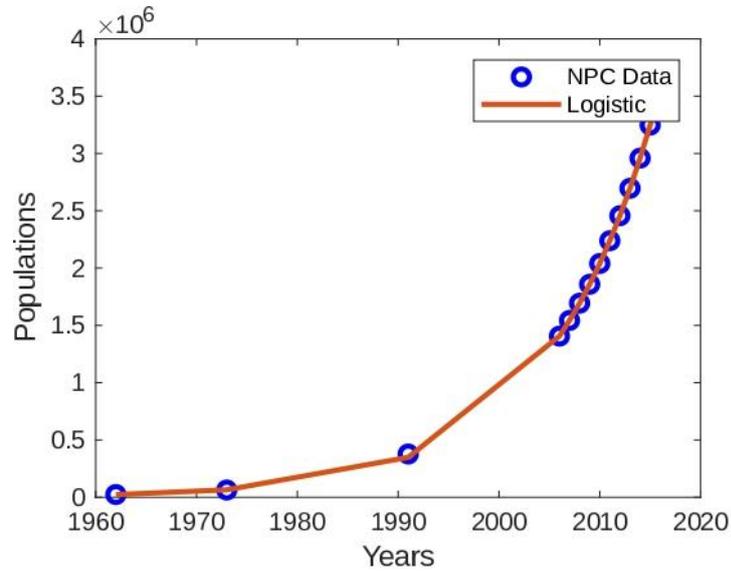**Figure 4.** Exponential Model simulation for 1962 to 2020

Figure 5. Logistic Model simulation for 1962 to 2020

3.1. Estimation of Parameters

We estimate the parameters involved for the logistic model and obtained the following values below. All codes were written in Matlab R2012b, and installed on a personal computer with Intel ® Core (TM) i5-2600 CPU@2.30 GHz and 8.00GB (7.78GB usable) RAM running on Windows 8.1 operating system.

Example 3.1: From [3], the population data published and gazette for Abuja is $P_0 = 1,543,293$, $P_1 = 1,693,706$ and $P_2 = 1,858,777$ for 2007, 2008, and 2009 respectively. Hence, by applying [2], we have

$$K = \frac{P_1(P_0 P_1 - 2P_0 P_2 + P_1 P_2)}{P_1^2 - P_0 P_2}, \quad A = \frac{K - P_0}{P_0}, \quad k = \frac{P_0(P_2 - P_1)}{P_2(P_1 - P_0)}$$

we can obtain the carrying capacity K and population growth rate r from solving the above system of equations as

$$K = 16,946,311,461, \quad r = 0.093009 \; A = 10979.62, k = 0.911184881$$

3.2. Error Analysis

Mean Absolute Percentage Error (MAPE): The Mean Absolute Percentage Error calculation is given by the formulae

$$\text{MAPE} = \frac{1}{N} \sum_{t=1}^{N} \left| \frac{A_t - F_t}{A_t} \right| \times 100\%$$

where A_t denotes actual value, F_t denotes forecast value and N is the number of observations.

Root Mean Square Error (RMSE): The Root Mean Square Error calculation is given by the formulae

$$\text{RMSE} = \sqrt{\sum_{t=1}^{N} \frac{(A_t - F_t)^2}{N}}$$

Where A_t denotes actual values, F_t denotes forecast values and N is the number of observations. According to [9] MAPE<10% is highly accurate, 10%−20% is good and

21%–50% is reasonable forecasting, while>50% is in accurate forecasting. However, the lower MAPE values are better because they signify smaller percentage errors.

R-Squared: The R-squared calculation is given by the formulae

$$\text{R} - \text{squared} = 1 - \frac{\sum(y_i - \hat{y})^2}{\sum(y_i - \bar{y})^2}$$

where y_i is the actual value and $\hat{y}$ is the predicted value. While $\bar{y}$ is the mean value. The closer the R^2 value is to 1, the better the correlation. The Table 1 below shows the results of the error analysis conducted on exponential and logistic population growth models using the simulated data of Abuja from1962 to 2016 shows the following results.

Table 1. Error Analysis of different Growth Models

Error Analysis	Exponential Model	Logistic Model
%MAPE	0.98	0.98
RMSE	7,811.07	7,817.07
R−Squared	0.776	0.776
Linear regression	y=60131x−1E+08	y=60129x−1E+08

3.3. Simulations

We present the population size simulation for Abuja using exponential model and logistic model. A sensitivity analysis model from 2007–2199 is also presented.

Table 2. Population simulation showing different models

Year	Time(t)	NPC Data	Exponential Model	Logistic Model	Sensitivity Model
2007	0	1,543,293	1,543,293	1,543,293	143527.0452
2008	1	1,693,706	1,693,707	1,693,706	157514.1336
2009	2	1,858,777	1,858,780	1,858,777	172864.0015
2010	3	2,039,937	2,039,942	2,039,934	189709.3651
2011	4	2,238,752	2,238,761	2,238,744	208195.8528
2012	5	2,456,945	2,456,957	2,456,928	228483.2589
2013	6	2,696,403	2,696,419	2,696,372	250746.9188
2014	7	2,959,199	2,959,219	2,959,147	275179.2161
2015	8	3,247,608	3,247,633	3,247,526	301991.2366
2016	9	3,564,126	3,564,157	3,564,002	331414.5804
2039	32		30,263,745	30,220,701	2805784.642
2060	53		213,357,987	210,817,405	19363987.38
2090	83		3,473,789,823	2,884,739,754	222633388.4
2107	100		16,882,256,619	8,461,080,785	394039070.4
2110	103		22,315,157,725	9,635,825,443	386620577.9
2150	143		920,858,296,579	16,640,479,158	27931803.21
2199	192		87,759,479,694,267	16,943,045,374	303716.9377

4. Discussion

It is clear that within the first few days of population increase, the exponential and logistic models predict identical numbers. Gradually, however, the numbers predicted by

the logistic model fall below those from the exponential model. By 2107, when the population is about 50% of carrying capacity, the value predicted for the exponential model has doubled that of the logistic model.

Table 2 shows that the Nigerian population from 1962 to 2016 increases from 24,314 people to 3,564,126 million. In Figure 1, it is observed that the populations over six decades from 1962 till 2016 increases. In Figure 5 the growth increases exponentially as the year increases in the first century from 2017 to 2039 with population increase from 3,911,313 million to 30,220,701 million people. It is observed that the population for the next century from 2107 to 2200 increases slightly and then tends to stability. Hence the population growth becomes slightly stable and then remains stable after 2200. In Figure 3 the growth sensitivity increases for the next three and half decades from 2015 to 2107.This means that the growth rate is still under some form of control or bound. After 2107 the growth sensitivity result shows that the population growth will become stable. From 2050 to 2150 the growth rate is out of control showing an increase in growth sensitivity which implies that everything should be done by the government both in policy and enforcement to ensure that the population growth is been checked early enough and controlled. This will bring the required stability of the population growth to an early time of the first century and not a later time of the second century. Finally, zero growth sensitivity is stable implying that in the second century the population growth of Nigeria beyond 2200 will attain stability.

5. Conclusions

In general, it can be seen that the total population of Abuja increases every year and the estimation result using logistic model has a relatively small error so that the estimation result of the population approaches the actual census result. Besides, the logistic population growth model was used to simulate the future population of Abuja up to 2199. As the population growth moves closer and closer to the carrying capacity, and there are no developmental plans and policies to engage and accommodate the population growth. In that case, there will be a crowding effect on the population density, which will bring about competition, resulting in hardship and hunger. Eventually, this will give birth to crises, insecurity, kidnapping, and other societal crimes. Regarding these factors mentioned, it is expedient that the government invest in massive agricultural reforms to accommodate the growing population, expand Abuja by developing its suburbs, and engage in massive reorientation of the populace on the dangers of uncontrolled births and the education of the girl child. A growing population when strategically harnessed can build a strong economy and the manpower that can build a strong army to defend its territory.

Conflicts of Interest: The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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