Scientific Publications

Journal of Social Mathematical & Human Engineering Sciences, 2023, 1, 610
www.scipublications.org/journal/index.php/jsmhes
DOI: 10.31586/jsmhes.2023.610

Review Article

Statistics of Electoral Systems and Methods of Election Manipulation

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Abstract: We study different types of electoral systems in the world, their features, advantages and problems. Legal election technologies for manipulating election results and ways to detect and eliminate the influence of such technologies are also considered. In addition, some results related to so-called "fair elections" are considered.

Keywords: Electoral System, Election Manipulation, Statistics

How to cite this paper:
H. Krykun, I., & S. Pavlov, M. (2023). Statistics of Electoral Systems and Methods of Election Manipulation. *Journal of Social Mathematical &Amp; Human Engineering Sciences*, *1*(1), 11–21. Retrieved from https://www.scipublications.com/journal/index.php/jsmhes/article/view/610

Academic Editor:
Houssam KHELALFA

Received: January 21, 2023
Accepted: March 28, 2023
Published: March 30, 2023

1. Introduction

Elections as a system of organization of representative power, expressing the interests of the majority of society, are an important tool of public administration. It is on the election of power that the state systems of all developed countries of the world are based, so according to the «Age of electoral democracy, 2021» [1], among 196 recognized countries, in 151 countries (and this number includes all highly developed ones) the government is elected through elections.

There are many publications on electoral issues; in particular J. Hodge & R. Klima in [2] described various electoral systems and modifications. Also they explained how «quota vote» works, and considered various results devoted to so called «fair election».

W. Wallis in [3] considered the theoretical foundations of electoral mathematics and the mathematical-statistical aspects of the voting organization process, fair and unfair vote. The book [3] also contains detailed discussions of various related topics, including methods of manipulating election results, amendments, and voting in small committees.

W. Gehrlein & D. Lepelley in [4] studied voting procedures based on the likelihood that paradoxical results may exist, such as the famous Condorcet paradox. There are well-known hypothetical examples of many different paradoxical election results, but in [4] authors examined the factors that are associated with the process by which voters form their preferences for candidates, which can reduce the possibility that such voting paradoxes will ever actually occur.

I. Krykun in a recent paper [5] proposed a new mathematical method for detection election fraud by estimating the parameters of some function from voting results and comparing these estimates with known theoretical parameter values.

[2-4] also contains a detailed description of the results of other authors.

2. Classification of electoral systems

In this section we describe the main electoral systems, their problems and inherent manipulations, and also provide possible ways to solve and avoid these problems.

2.1. Types of electoral systems

Although the terminology and classification of electoral systems may differ, we give a classification according to authoritative sources «IDEA» [6] and «ACE» [7]. So, there are 9 main electoral systems which belong to the 3 main families of electoral systems: plurality/majority, proportional and mixed (Figure 1).

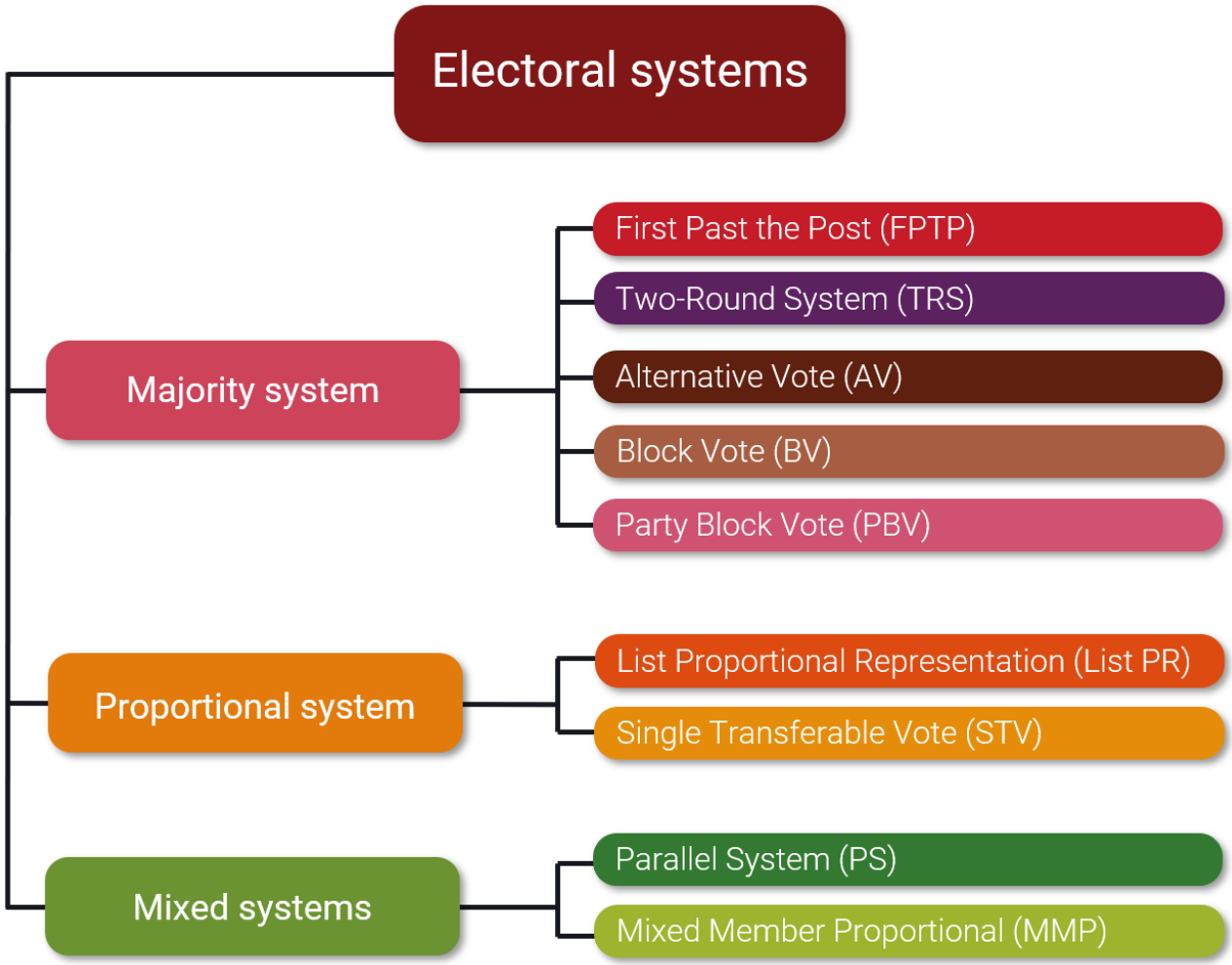

Figure 1. Classification of electoral systems [7].

2.2. Description of electoral systems

Majority electoral systems – the winners are the candidates who received the majority of votes.

Subtypes of majority systems include:

- First Past The Post (FPTP) – voters can cast a vote to only one candidate on the list; the candidate with the most votes wins.
- Two-Round System (TRS) – elections are held in two rounds, in the first round several (usually two) candidates with the highest number of votes are determined, these candidates advance to the second round. The winner of the election is the candidate with the most votes in the second round of voting. The second round is not held if there is a candidate in the first round who received more than 50% of the vote.

- Alternative Vote (AV) – voters can vote for several candidates, but at the same time rank candidates in the order of their choice: voter gives a 1 to their most preferred candidate, a 2 to their second most preferred, and so on.
- Block Vote (BV) – used when multiple candidates need to be selected. Voters have as many votes as there are candidates to choose.
- Party Block Vote (PBV) – voters have one vote, and vote not for individual candidates, but for lists of candidates. As in the FPTP, the list may not gain an absolute majority. All seats are occupied by candidates from the list that received the largest number of votes.

Proportional electoral systems – parties get seats in proportion to the percentage of votes they received. The proportional system requires the use of constituencies with multiple seats (since it is impossible to divide a single seat proportionally).

- List Proportional Representation (List PR) – provides that each party nominates a list of candidates in each multi-member constituency. Voters vote for the party, and parties get seats in proportion to their share of votes in the constituency. Winning candidates are selected from the lists in the order of their position on the lists.
- Single Transferable Vote (STV) is voting in a multi-member constituency where voters rank candidates on ballots in preferred order in the same order as in the alternative voting system (each voter has one vote). After counting the total number of votes, a quota is set. Usually use the Droop quota [8], calculated by the formula:

$$\text{Quota} = \frac{\text{votes}}{\text{seats} + 1} + 1,$$

where «votes» is the total number of votes cast;
«seats» – the number of vacant seats for elected positions.

After the voting, the voting positions are distributed in proportion to the votes received (with a rather complicated mechanism for redistributing votes received above the quota, a detailed description of the procedure can be found in [8]).

Mixed electoral systems – to eliminate the shortcomings of both systems under consideration, the so-called mixed (or semi-proportional) electoral system is called for, in which some deputies are elected by a majoritarian system, and the other part – by proportional system.

- Mixed Member Proportional (MMP) voting is a mixed electoral system in which voters receive two votes: one vote to select a representative in a single-member constituency, and one vote to select a political party in a multi-member constituency.
- Parallel System (PS) is an electoral system that uses components of both proportional and majoritarian systems. It differs from voting under a mixed system in that the components of a parallel system do not compensate for any disproportion. In a parallel system, each voter can receive either one ballot used to vote for both the candidate and his party, or two separate ballots, one for voting for candidates for deputies under the majoritarian system and one for voting for political parties under the proportional system.

In addition to the aforementioned electoral systems, there are several other systems that do not fit into any of the categories mentioned above. These systems include [5]: Single Non-Transferable Vote (SNTV), Limited Vote (LV), and Borda's Count (BC). These three systems are a cross between how votes are transformed into seats in power in the proportional and majority system.

2.3. Statistics of electoral systems

Among the 218 (full and partially recognized) countries and territories of the world, the majoritarian system accounts for 40.4% (88 countries), 37.6% is proportional (82 countries), 14.7% (32 countries) is mixed, 3.2% (7 countries) have other electoral systems,

1 country is in the process of transition from one system to another, and 3.7% (8 countries) do not use elections to elect a national legislative body [9].

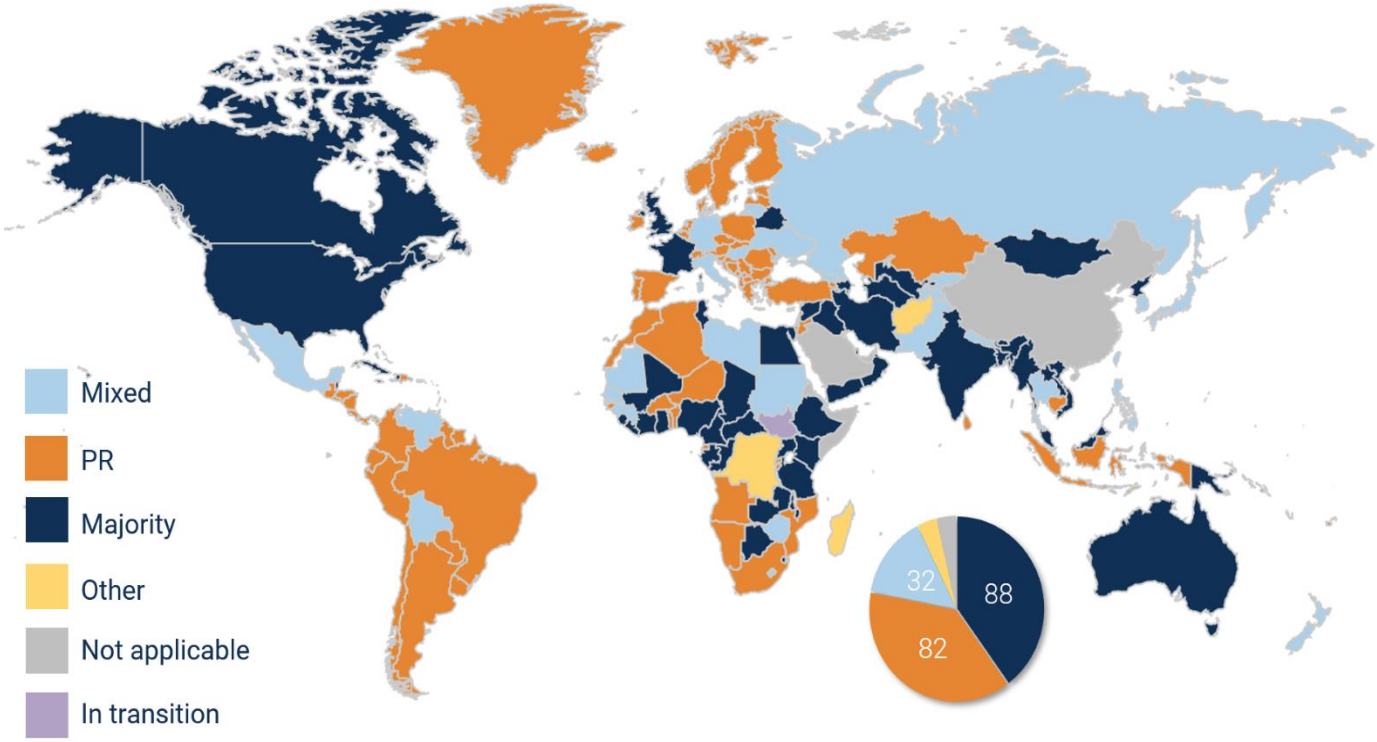

Figure 2. Types of electoral systems (in the last election, 2022 or before). Data for 218 countries and territories [9].

3. The main methods of elections manipulation

Despite the fact that in order to eliminate situations of the «unfair» outcome of the will, mankind has come up with many of the types of electoral systems described above, a lot of manipulative technologies and techniques have been invented that exploit the vulnerabilities of a particular electoral system and make it possible to influence the outcome of the elections. Moreover, we are not talking about outright falsifications and falsifications of the results (which became famous in particular for the USSR and the modern Russian Federation), and not about dubious technologies like «black PR», but about quite legal methods (which, perhaps, can be called unethical).

3.1. Manipulations with district boundaries

Let's start with such a technique as «Gerrymandering» [10], [11]. This technique is one of the most illustrative when it comes to election manipulation.

Let us give an example of gerrymandering. Imagine that there are 50 voters and two candidates for the same position, we will designate them «red» and «blue». Elections are held and 60% of voters vote for the «blue» candidate, and 40% for the «red» (Figure 3 below).

With this distribution, the «blue» candidate will win, since he will receive a majority of votes. But under a different voting system ((FPTP), in this case there are several constituencies) under certain circumstances a «red» candidate can win. Let's change the rules: we will break all voters into several districts and recount the votes in each of them separately, and the winner in the constituency will be the candidate who won in most districts. If we divide the constituencies like on Figure 4 below, then the «blue» candidate also will win.

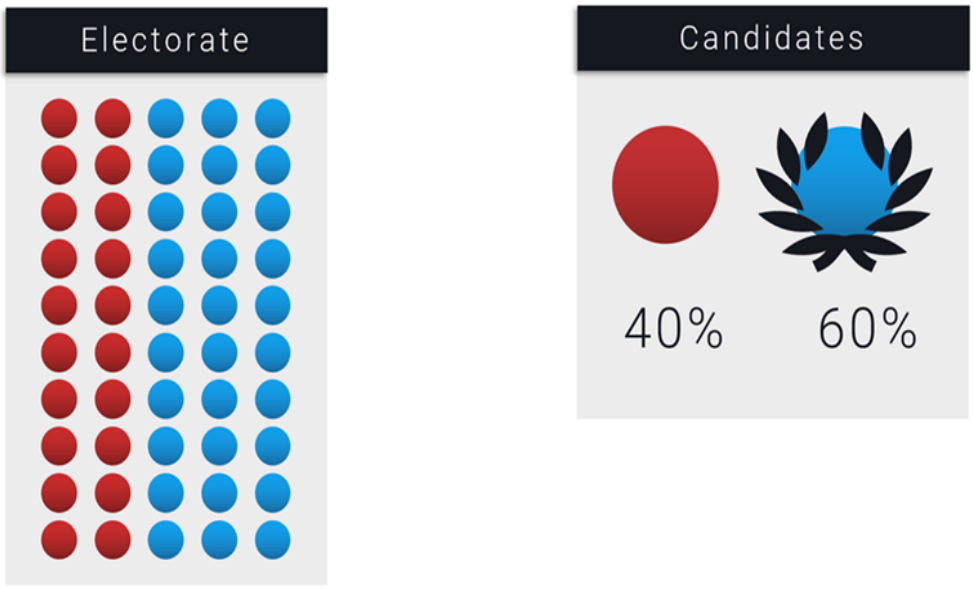

Figure 3. Distribution of votes and the winner when voting under the majority system of simple majority.

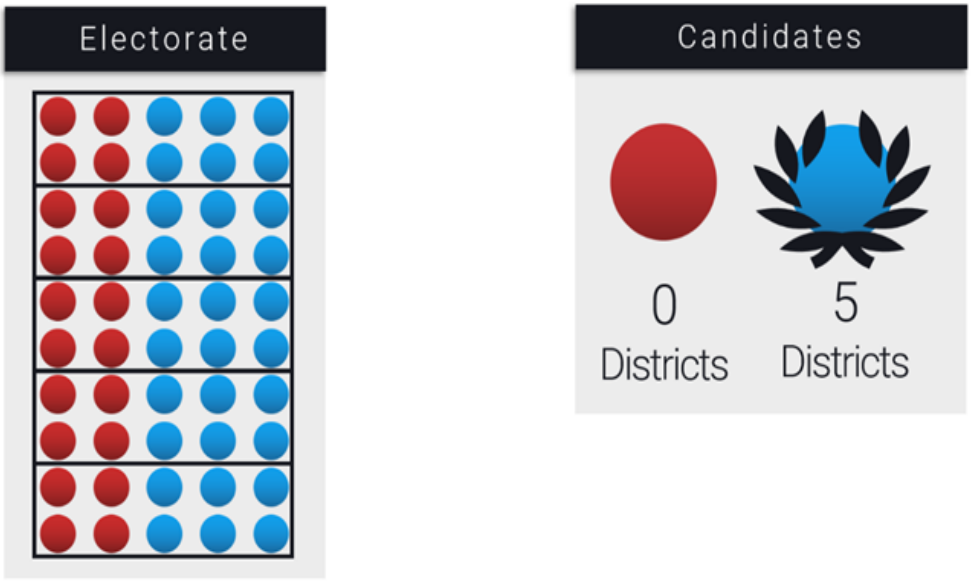

Figure 4. Distribution of electorate and the winner in a situation where the winner is the candidate who won in most constituencies.

In case of such distribution as on Figure 4 «blue» candidate won all five districts, but it is possible to divide electorate in a completely different way, for example, as shown below (Figure 5).

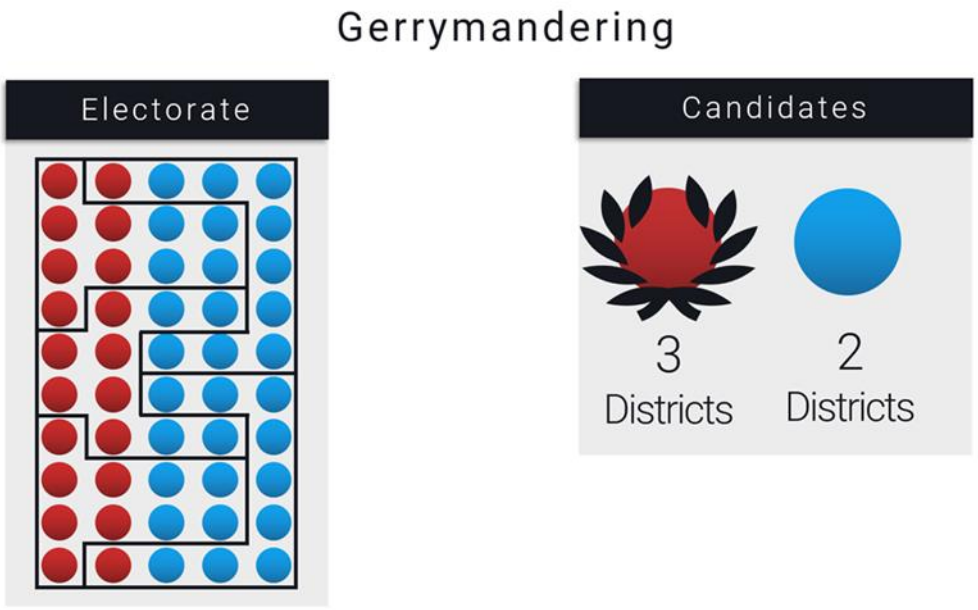

Figure 5. "Gerrymandering" is a manipulation of constituency boundaries, which leads to the victory of the «red» candidate in a situation where the winner of the election is the one who won in most districts. As before, there are ten people in each constituency, but now the «blue» candidate has a majority only in two districts but the «red» one has a majority in three districts, so the «red» candidate wins.

The word "gerrymander" arose in 1812 when a caricaturist depicted one of the constituencies of the state of Massachusetts (USA) in the form of a salamander (Figure 6) [11], and the publisher of the newspaper called this drawing "Gerrymander", after the last name of the governor of the state, Elbridge Thomas Gerry, who first applied this system. Thus, the term "gerrymander" literally means "Gerry's salamander".

Figure 6. "Gerrymander" caricature [11].

When T. Gerry was governor of Massachusetts in 1812, he signed a law allowing his party to establish constituencies in the state [10]. Through manipulation of constituencies' boundaries, Gov. T. Gerry achieved that in the state Senate elections, his Democratic-Republican party won 29 seats out of 40, while the Federalist party won only 11 seats. This result was obtained despite the fact that the governor's opponents (the Federalists) received the majority of the vote.

In fact, such a distribution of voters by constituency is necessary in order to equalize the constituencies, because people are gradually moving, and new districts are being built, so the boundaries of the constituencies should be periodically changed.

Such election manipulations occur in majoritarian systems when a country is divided into several constituencies, and the candidate with the most votes wins the constituency (usually FPTP) [12].

3.2. Manipulations with «technical» candidates

Gerrymandering will not be applicable in the overall vote count, but there is another «technique» that can be used in almost all elections. It's about vote splitting. «Weak» candidates who have no real chance for win, pull away some of the votes, thereby weakening the positions of the favourites and preventing them from winning [2].

Such election manipulations are inherent in following electoral systems as FPTP, TRS, BV, PBV, List PR [13]. The vote splitting rarely occurs when the selected electoral system uses ranked ballots (AV) and a pairwise counting method such as the Condorcet method [2], [3].

A variation of the effect of vote splitting is the effect of the «spoiler candidate» [14]. As a rule, the distribution of votes is not intentional, but it is possible to make a division and artificially by introducing so-called «technical candidates» or «spoiler candidates» into the election, these are election participants who do not aim to win. They often copy the symbols and program of other candidates to «delay» the votes of their voters. The problem is that you can't ban someone from participating in elections if they have fulfilled all the requirements. Thus, it is almost impossible to fight technical candidates, although because of them, in fact, not the most popular candidate can win.

In addition to technical candidates, there are also «clone» candidates whose goal, the same as that of technical candidates, is to «delay» the more votes and weaken the position of other designated candidates whom they «clone». The «cloning» technology involves registering a candidate with the same last name and initials as a competitor in order to spray these votes between «namesakes» and thus «bite off» a little from a competitor. After all, with similar names, it is easier for the voter to make a mistake. For example, more than 90 thousand voters voted for «clone» candidates in the last parliamentary elections in Ukraine in 2019 [15].

3.3. Voter turnout manipulations

The results of elections depend not only on random and variable support for candidates but also to a certain extent on the – also random – turnout of voters, which, in the case of approximately equal support for candidates, becomes decisive.

There are some relatively legal election technologies pointed on voter turnout – such as "black PR", motivating "own" voters to visit the polling stations, demotivating "other" voters, or presenting "own" candidate as the "lesser evil" for those voters who are undecided.

The implementation of such election technologies (i.e. the motivation of "own" voters and demotivation of "other" voters through social networks and contextual political advertising as a tool for such motivation in particular) could be observed in recent election processes even in such stable democracies as Great Britain (the 2016 United Kingdom European Union withdrawal membership referendum; the technologies used on this

referendum are described in the film based on true stories "Brexit: The Uncivil War") and the USA (the 2016 United States presidential election when Donald Trump was elected President; the technologies used on these both campaign are described in the film "The Great Hack").

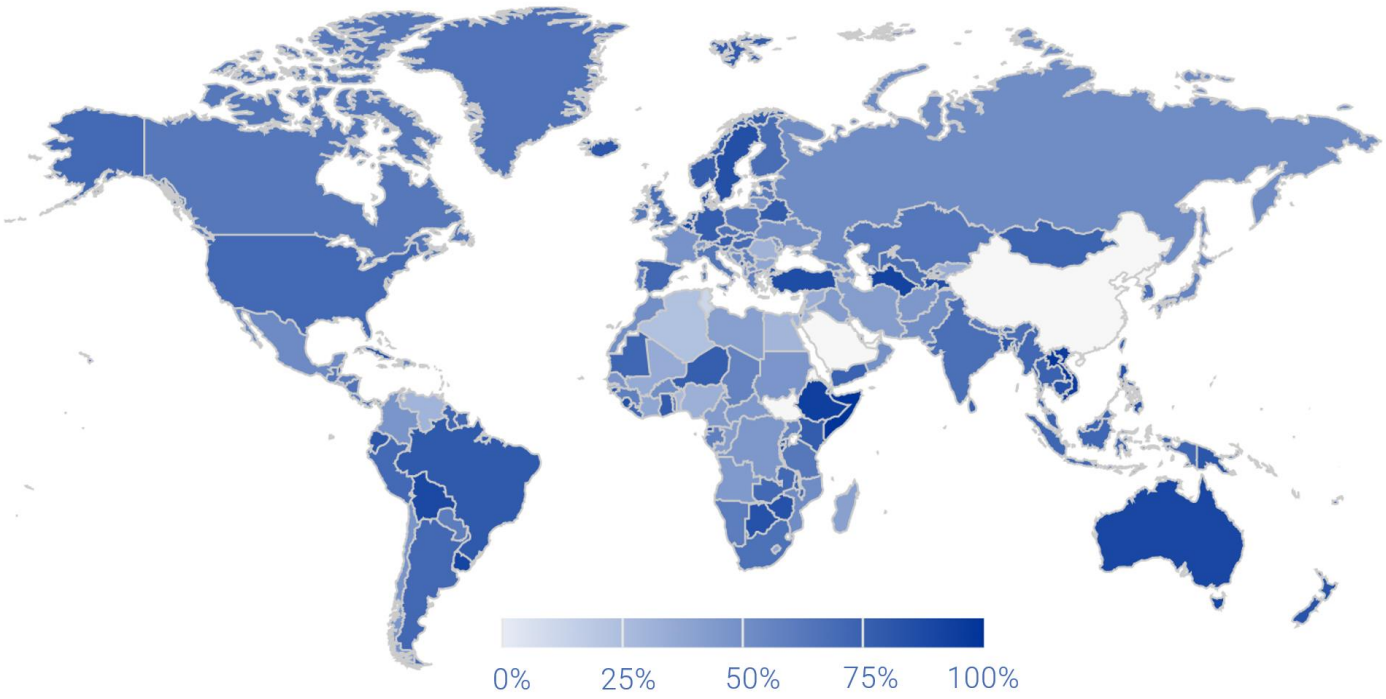

Figure 7. Voter turnout in the last (2022 or earlier) parliamentary election. Data for 201 countries and territories [16].

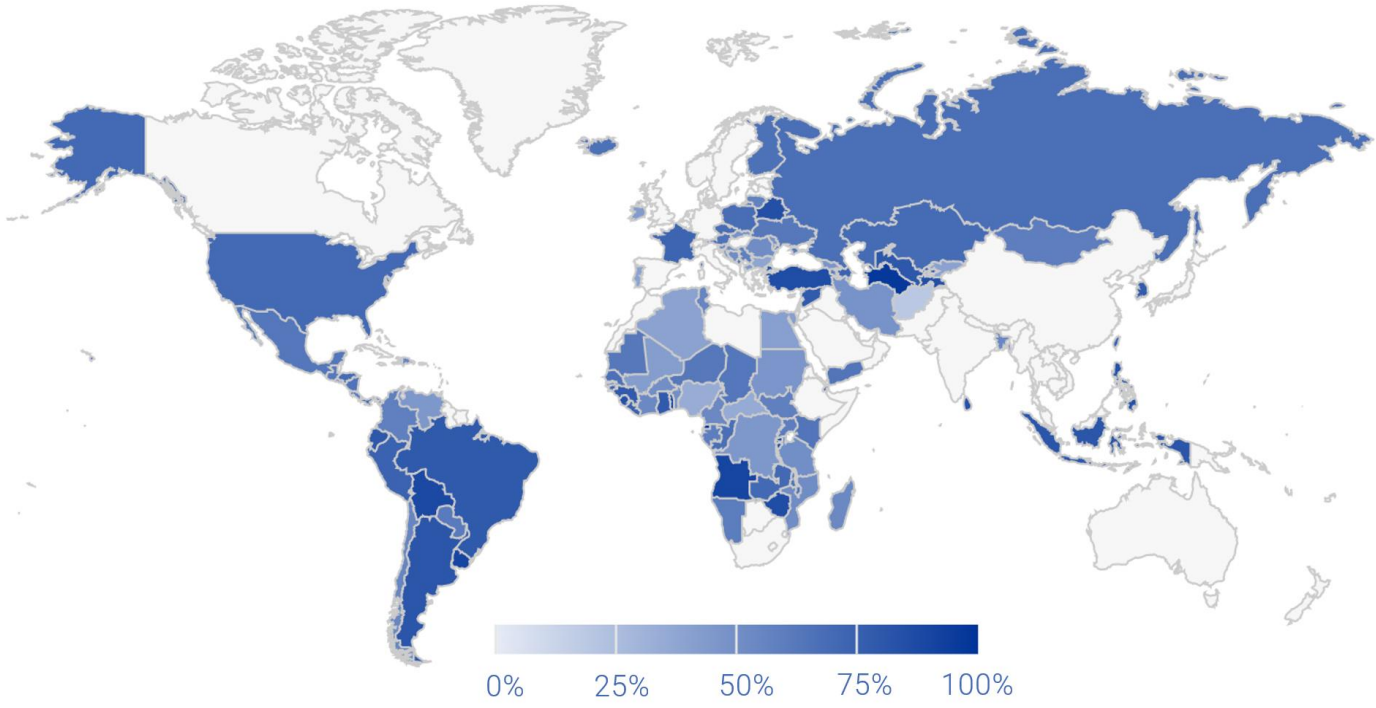

Figure 8. Voter turnout in the last (2022 or earlier) presidential election. Data for 114 countries and territories [16].

3.4. Advantages and problems other electoral systems

One way to avoid the division of votes is preferential voting (AV, STV). This is an electoral system that indicates not only the priority candidate, but also the degree of priority of other candidates (point or place) [4]. Under such a system, it is the one who, on average, ranks highest in the ratings of all voters, wins.

Such systems are used in Australia, Malta, New Zealand, Ireland, India, and many cities in other countries in local government elections. Due to the fact that such a voting system provides a lot of information about the preferences of voters, it becomes possible to choose exactly the candidate who is the most authoritative for voters.

Although such a system seems perfect, it also has its drawbacks. For example, consider the system of counting votes using the Borda method [2]. According to this method, the voting results are expressed in the form of the sum of points earned by each of the candidates. If in elections there are n candidates, each voter ranks all candidates in descending order: for the best candidate is given n points, for the second candidate is given $(n-1)$ points, etc. (for the worst candidate is given 1 point), all points earned by the candidate are summed up. The winner of the election is the candidate who earned more points than every other candidate. The problem is in the practical application of the method, because the Condorcet paradox often arises when the preferences of opposite groups of voters come into conflict with each other [3].

4. The Arrow's theorem and the Condorcet principle as paradoxes of electoral systems

4.1. The Arrow's theorem

There are many others methods of selecting candidates, but every method has one property: trying to eliminate one drawback – inevitably another is created. The fact that it is impossible to create an ideal electoral system derives from the «Arrow's impossibility theorem» [2], [3].It states that, paradoxically, there is no electoral system without flaws, and cannot exist at all. This system is based on four basic principles of fair elections:

- **Universality** – electoral systems should not impose any conditions, other than transitivity, on how voters can streamline candidates in elections.
- **Nondictatorship** – all voters are equal, and there is no voter whose vote would be more important than another.
- **Independence of Irrelevant Alternatives**, in particular from «clone» candidates. The operation of the electoral system should not be affected by the presence or absence of third-party candidates.
- **Unanimity** – «If there is a pair of candidates in the election, A and B, such that each voter prefers A to candidate B, then in the final public order of preferences A should go ahead of B». That is if the majority of voters in the pair of candidates A, B prefer candidate A, then in the overall final result A should win.

K. Arrow proved that if there are more than two candidates, then whatever system people come up with, at least one of the principles will be violated. That is, if you have to choose more than two candidates – there will be no ideal elections and an ideal democracy, any electoral system will be flawed and it will not be possible to fix it.

4.2. The Condorcet principle and paradox

In addition to the problems described above, another controversial phenomenon known as Condorcet's Paradox [4] may occur in elections.

According to the **Condorcet principle**, in order to compare the possible alternatives to be chosen, it is necessary that each voter ranks all alternatives in order of their preference. Then they compare each alternative in pairs with all the others, and determine to whom the majority of the voters, in a pair of alternatives, prefers. Thus, you can compare any candidates.

For example, let's suppose that there are three candidates A, B, and C. We denote A > B > C if the voter prefers candidate A more than B, and candidate B is preferable to C.

Let the 60 voters express their preferences as follows:

23 persons: A > C > B
19 persons: B > C > A
16 persons: C > B > A
2 persons: C > A > B

So 25 persons vote that A > B, and 35 persons vote that B > A, therefore according to Condorcet principle, the majority opinion is that B is better than A. Further, 23 persons vote that A > C and 37 persons vote C > A. Thus in a pair of A, C, candidate C will win. Similarly (19 votes for B > C, 41 votes for C > B) candidate C is more desirable than B.

According to the Condorcet principle, the public preferences look like this: C > B, B > A, C > A. These three relations can be combined into one C > B > A, where the winner will be C.

If in this case, the system of voting of the relative majority is chosen, then the following results will occur: candidate A will earn 23 votes, candidate B will earn 19 votes, and candidate C will earn 18 votes. So if using this system, candidate A will win.

If voting is under the absolute majority system, candidates A and B will advance to the second round of elections, where candidate A will earn 25 votes but candidate B will earn 35 votes and will win.

We conclude that the election rules determine the winner, and the vote winners can be different for the same electorate under different voting rules.

Condorcet's paradox is that if the votes are equally divided, such as this:

1 person: A > B > C
1 person: C > A > B
1 person: B > C > A

then, based on the results of the vote, three statements will stand out: B > C, C > A, A > B. Together, these claims will be contradictory. In this case, it turns out to be impossible to make any agreed decision and determine the will of the majority.

5. Conclusions and prospects for further research

Within the framework of this work, the classification of electoral systems was considered, and the statistics of electoral systems in the world according to these main types (majoritarian, proportional, mixed) were given. The vulnerability of electoral systems to the most typical manipulations is considered. The chapter is devoted to Arrow's theorem and Condorcet's paradox. Prospects for further research consist of the analysis and study of other electoral systems of the world, including "exotic" ones, and the identification of problems in them. It is planned to investigate the factors in electoral systems that lead to the problems described above in order to reduce the likelihood of their occurrence. In future works, it is planned to find such an electoral system in which it will be possible to eliminate the shortcomings as much as possible.

The study of world electoral systems and the search for ways to detect and to avoid possible election manipulations will be enhanced using results and methods of recent studies of applications of random processes [17, 18, 19, 20, 21, 22, 23, 24, 25] and computer technologies [24, 26, 27].

Author Contributions: All authors have read and agreed to the published version of the manuscript.

Funding: This research received no external funding.

Acknowledgments: The authors express their heartfelt gratitude to the brave soldiers of the Ukrainian Armed Forces who protect the lives of the authors and their families from Russian bloody murderers since 2014.

Conflicts of Interest: The authors declare no conflict of interest.

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