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Article

The Calculation of the Binding Energy of the Exciton Moving in a Two dimensional Semiconductor Quantum Well

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Abstract: With respect to the exciton moving in the two-dimensional quantum well, the paper presents a scheme which can rigorously calculate out the binding energy of the exciton in the two-dimensional semiconductor quantum well by simply using the relation $|z_e - z_h| = \rho \tan\alpha$, which is much simpler than the complex calculation of Ref.[1-2]. Concerning the calculation result eq.(13), the paper discusses the results for two significant cases of $|z_e - z_h| \ll \rho$ and $|z_e - z_h| \to \infty$.

Keywords: Exciton, Binding Energy, TheVariational Method

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1. Introduction

Since the concept of superlattice was proposed in 1970 years, The superlattice materials have been widely applied in mass productions of various semiconductor elements and instruments. The superlatice is constructed by quantum wells, it is the so-called the project of energy bands. The binding energy is an elementary physical quantity for an exciton, Calculating out the binding energy of the exciton moving in a quantum well is significant for applications of superlattice materials. On the basis of some related studies[1-3], the paper will discuss the calculation method of the binding energy of the exciton moving in a two-dimensional quantum well.

2. The Wave Function of the Exciton

With respect to the motion of the exciton, the paper will not consider those more complex cases. Supposing the direction of the crystal growth is along the z axis, under the action of the external electric field along the z direction, the exciton moves in the plane which is vertical to the direction of the crystal growth, thus, the wavefunction of the exciton moving in the quantum well is written[1,2].

$$\Psi_{exc}(\vec{r}_e, \vec{r}_h) = F(z_e, z_h) \cdot \exp(-\rho/\lambda), \tag{1}$$

where $\exp(-\rho/\lambda)$ describes the binding exciton in the plane, λ notes the track radius of the exciton in the plane. For the general cases, we merely need to consider the coulomb potential of the system, $F(z_e, z_h)$ can be therefore written

$$F(z_e, z_h) = N \cdot f(z_e) \cdot f(z_h), \tag{2}$$

where $f(z_e)(f(z_h))$ is the wave function of the ground state of the electron(hole), which can be obtained from solving the Schodinger's equation. If the electron is exerted by the external electric fields in the quantum well, from the Schodinger's equation it obtains.

$$f(z_e)=\begin{cases} A_e\exp[(q_e-\beta_e)\cdot(z_e+\tfrac{1}{2}L_e)],(z_e<-\tfrac{1}{2}L_e) \\ B_e Cos(k_e\cdot z_e),(-\tfrac{1}{2}L_e<z_e<\tfrac{1}{2}L_e) \\ C_e\exp[(-(q_e+\beta_e)\cdot(z_e-\tfrac{1}{2}L_e),(z_e>\tfrac{1}{2}L_e) \end{cases} . \quad (3)$$

In eq.(3), substituting β_e with $-\beta_h$; A_e with A_h , and L_e with L_h, thus, $f(z_h)$ can be obtained from eq.(3). Where $\vec{k}_e,\vec{q}_e(\vec{k}_h,\vec{q}_h)$ are the ground state vectors of the electron(hole) in the quantum well, $\vec{q}_e$ and $\vec{q}_h$ are relative to the height of the potential barrier; L is the width of the quantum well; $\beta_e(\beta_h)$ is a parameter which is relative to the electron(hole) and the external electric field as well; and $\beta_e=\frac{2\pi\cdot\sqrt{2m_e\cdot\varepsilon_{ef}\cdot e}}{h}$, $\beta_h=\frac{2\pi\cdot\sqrt{2m_h\cdot\varepsilon_{ef}\cdot e}}{h}$, where ε_{ef} notes the intensity of the external electric field.

Therefore, using the boundary conditions, it obtains[2]

$$F(z_e,z_h)=N(\beta_e)N(\beta_h)Cos\frac{z_e}{L_e}\pi\cdot\exp[-\beta_e\cdot(z_e+\frac{1}{2}L_e)]\cdot Cos\frac{z_h}{L_h}\pi\cdot\exp[-\beta_h\cdot(z_h+\frac{1}{2}L_h)] . \quad (4)$$

In terms of the condition of normalization, i*t calculates out*

$$N^2(\beta)=\frac{4\beta}{L[1-\exp(-2\beta)]}\cdot[1+(\frac{\beta}{\pi})^2] . \quad (5)$$

Through the normalization, the $\exp(-\rho/\lambda)$ part in the wave function of the exciton can be written

$$\Phi_{e-h}=(\frac{2}{\pi})^{1/2}\cdot\frac{1}{\lambda}\cdot\exp(-\rho/\lambda) . \quad (6)$$

3. The Binding Energy of the Exciton in the Quantum Well

In consideration of the exciton in the quantum well, its binding energy consists of two parts, one of them is the kinematical energy of their relative motion in the plane which is vertical to the direction of the crystal growth, and the other is their coulomb potential, namely,

$$E_B=E_{kr}+E_{pr} , \quad (7)$$

where E_{kr} and E_{pr} respectively note the relative kinematical energy and the coulomb potential of the exciton, they are respectively calculated by

$$E_{kr} = \langle \Phi_{e-h} | H_{ke-h} | \Phi_{e-h} \rangle \ , \tag{8}$$

and

$$E_{pr} = \langle \Psi_{exc} | V_{e-h} | \Psi_{exc} \rangle \ . \tag{9}$$

Substituting eq.(6) and H_{ke-h} into eq.(8), it obtains

$$E_{kr} = \langle \Phi_{e-h} | H_{ke-h} | \Phi_{e-h} \rangle = \frac{\hbar^2}{8\pi^2 \mu \lambda^2} \ . \tag{10}$$

Where μ is the equivalent mass of the electron and the hole. Because the direction of the crystal growth is along z axis, therefore,

$$E_{pr} = \frac{2}{\pi\lambda^2} \cdot (-\frac{e^2}{\varepsilon}) \cdot \int_0^{\infty} \int_{-L_e/2}^{L_e/2} \int_{-L_h/2}^{L_h/2} \begin{array}{l} N^2(\beta_e) \cdot N^2(\beta_h) \cdot Cos^2 \frac{\pi \cdot z_e}{L_e} \cdot \exp[-2\beta_e \cdot (z_e + \frac{1}{2} L_e)] \cdot \\ Cos^2 \frac{\pi \cdot z_h}{L_h} \cdot \exp[-2\beta_h \cdot (z_h + \frac{1}{2} L_h)] \cdot \\ \frac{\rho \exp(-2\rho/\lambda)}{[|z_e - z_h|^2 + \rho^2]^{1/2}} d\theta d\rho dz_e dz_h \end{array} \ , \tag{11}$$

where ε notes the dielectric constant of the semiconductor material, $L_e(L_h)$ is the approximate effective well-width of the electron(hole).

In eq.(11), because $|z_e - z_h|$ can be replaced by $\rho \cdot \tan\alpha$, so it does not influence the calculations of the integrals about dz_e and dz_h. According to the normalization of $F(z_e, z_h)$, the binding energy of the exciton is estimated by

$$E_B = \frac{\hbar^2}{8\pi^2 \mu \lambda^2} - \frac{4e^2}{\varepsilon \lambda^2} \int_0^{\infty} \frac{\rho \cdot \exp(-2\rho/\lambda)}{[|z_e - z_h|^2 + \rho^2]^{1/2}} d\rho \ , \tag{12}$$

with respect to the integral in eq.(12), $|z_e - z_h| = \rho \cdot \tan\alpha$, α is the angle clipped by ρ and $[|z_e - z_h|^2 + \rho^2]^{1/2}$, therefore, eq.(12) becomes

$$E_B = \frac{\hbar^2}{8\pi^2 \mu \lambda^2} - \frac{4e^2}{\varepsilon \lambda^2} \int_0^{\infty} \exp(-2\rho/\lambda) d\rho = \frac{\hbar^2}{8\pi^2 \mu \lambda^2} - \frac{2e^2}{\varepsilon \lambda} \cdot \frac{1}{[1 + \tan^2 \alpha]^{1/2}} \ . \tag{13}$$

Concerning the result of binding energy of the exciton eq.(13), two discussions are given in the following:

(1) If the exciton in the plane, and $|z_e - z_h| \ll \rho$, thus, $\tan\alpha \to 0$, eq.(13) becomes

$$E_B = \frac{\hbar^2}{8\pi^2 \mu\lambda^2} - \frac{4e^2}{\varepsilon\lambda^2}\int_0^\infty \exp(-{}^{2\rho}\!/\!_{\lambda})d\rho = \frac{\hbar^2}{8\pi^2\mu\lambda^2} - \frac{2e^2}{\varepsilon\lambda} \ . \tag{14}$$

Recognizing the parameter λ as a variable, using the variational method, from eq.(14) it obtains the binding energy of the exciton

$$E_{B\min} = -\frac{8\pi^2\mu e^4}{\varepsilon^2 h^2} \ . \tag{15}$$

It is well-known that the ground state energy of the hydrogen atom is given by

$$E_1 = -\frac{2\pi^2 \mu_{e-p} e_s^{\ 4}}{h^2} \ , \tag{16}$$

in SI unit, $e_s = e\cdot(4\pi\varepsilon_0)^{1/2}$, thus,

$$E_1 = -\frac{2\pi^2\mu_{e-p}e_s^{\ 4}}{h^2} = -\frac{\mu_{e-p}e^4}{8\varepsilon_0^{\ 2}h^2} \ . \tag{17}$$

Comparing eq.(15) with eq.(16), it gets:

$$E_{B\min} = (\frac{8\pi}{\varepsilon_r})^2 \times \frac{\mu}{\mu_{e-p}} E_1 \ . \tag{18}$$

For example, with respect to the semiconductor Ge, its ε_r is about 16, therefore,

$$E_{B\min} \approx 2.47 \times \frac{\mu}{\mu_{e-p}} E_1 \ . \tag{19}$$

Where μ_{e-p} is the equivalent mass of the electron and the proton, in general, $\frac{\mu}{\mu_{e-p}} < 1$. Therefore, when $|z_e - z_h| \ll \rho$, the exciton similar to a system which consists of an electron and a core with a positive charge, like a hydrogen atom, its binding energy small differs from the ground state energy of the hydrogen atom, this is in accord with the theory of quantum mechanics.

(2) If $|z_e - z_h| \to \infty$, λ is a parameter, using the variatinal method, from eq.(13), it obtains

$$E_{B\min} = \frac{4\pi^2 \mu e^4}{\varepsilon^2 h^2}\left[\frac{1}{1+\tan^2\alpha} - \frac{4}{\sqrt{1+\tan^2\alpha}}\right]. \quad (20)$$

Therefore, when $|z_e - z_h| \to \infty$, thus $\tan\alpha \to \infty$, $E_{B\min} = 0$. It demonstrates that there is no binding between the electron and the hole, they become two lonely particles.

4. Conclusion

The paper rigorously calculated out the bnding energy of the exciton moving in the quantum well by simply using the relation of $|z_e - z_h| = \rho \tan\alpha$ to calculate out the integral $\int_0^\infty \frac{\rho \cdot \exp(-{2\rho}/{\lambda})}{[|z_e - z_h|^2 + \rho^2]^{1/2}} d\rho$, the present calculation is rather simple than the complex calculations in the Ref.[1-2]. In the discussions about the result eq.(13) in case of $|z_e - z_h| \ll \rho$, from the rigorously and carefully calculation, the paper got the result of the binding energy of the exciton which is approximate to the ground state energy of the hydrogen atom. Moreover, in the case of $|z_e - z_h| \to \infty$ in the result, from eq.(20) it gets $E_{B\min} = 0$.

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