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Article

Charged Stellar Model with Generalized Chaplygin Equation of State Consistent with Observational Data

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Abstract: In this paper, we found a new model for a compact star with charged anisotropic matter distribution considering the generalized Chaplygin equation of state. The Einstein-Maxwell field equations have been solved with a particular form of metric potential and electric field intensity. The plots show that physical variables such as radial pressure, energy density, charge density, anisotropy, radial speed sound, and the mass are fully well defined and are regular in the star's interior. We obtained models consistent with stellar objects such as GJ 832, LHS 43, SAO 81292, GJ 380, GJ 412, and SAO 62377.

Keywords: Einstein-Maxwell field equations; Chaplygin equation of state; Electric field intensity; Metric potential; Radial pressure; Anisotropy

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1. Introduction

The study and description of static fluid spheres is an interesting area of research and one of great relevance in astrophysics due to the formulation of the general theory of relativity [1, 2]. One of the most important issues in general relativity is finding exact solutions to Einstein's field equations to propose physical models of compact stars as suggested by Delgaty and Lake [3] who constructed several analytic solutions that describe static perfect fluid and satisfy all the necessary conditions to be physically acceptable [3]. These exact solutions have also made it possible the way to study cosmic censorship and analyze the formation of naked singularities [4].

In the construction of theoretical models of stellar objects, the research of Schwarzschild [5], Tolman [6], and Oppenheimer and Volkoff [7] is very important to be considered. Schwarzschild [5] found analytical solutions that allowed the description of a star with uniform density, Tolman [6] developed a method to find solutions for static spheres of fluid, and Oppenheimer and Volkoff [7] used Tolman's solutions to study the gravitational balance of neutron stars. It is important to mention that Chandrasekhar's contributions [8] in the model production of white dwarfs and the presence of relativistic effects and the works of Baade and Zwicky [9] fully propose the main physical concepts of neutron stars and also identify astronomic dense objects known as supernovas.

The presence of the electric field can modify the values for surface redshift, luminosity, density and maximum mass for stars. Bekenstein [10] considered that the gravitational attraction may be balanced by electrostatic repulsion due to electric charge and pressure gradient. Komathiraj and Maharaj [11] obtained new classes of exact solutions to the Einstein-Maxwell system of equations for a charged sphere with a particular choice of one of the metric potentials. Ivanov [12] has studied and developed a wide variety of charged stellar models. More recently, Malaver and Kasmaei [13] proposed a model of charged anisotropic matter with the nonlinear equation of state.

It is well known the fact that the anisotropy plays a significant role in the studies of relativistic stellar objects [14-26]. The existence of a solid core, the presence of type 3A superfluid [27], a magnetic field, a mixture of two fluids, a pion condensation, and an electric field [28] are the most important reasonable facts that explain the presence of anisotropy. Bowers and Liang [14] generalized the equation of hydrostatic equilibrium for the case of local anisotropy.

Many researchers have used a variety of analytical methods to try to obtain exact solutions of the Einstein-Maxwell field equations for anisotropic relativistic stars. It is very important to mention that the contributions of Komathiraj and Maharaj [11], Thirukkanesh and Maharaj [30], Maharaj et al.[31], Thirukkanesh and Ragel [32,33], Feroze and Siddiqui [34,35], Sunzu et al.[36], Pant et al. [37] and Malaver [38-41] need to be considered in this field of research study. These studies suggest that the Einstein-Maxwell field equations are very important in the description of ultracompact objects.

The development of theoretical models of stellar structures can consider several forms of equations of state [42]. Komathiraj and Maharaj [43], Malaver [44], Bombaci [45], Thirukkanesh and Maharaj [30], Dey et al. [46] and Usov [28] assume a linear equation of state for quark stars. Feroze and Siddiqui [34] considered a quadratic equation of state for the matter distribution and specified particular forms for the gravitational potential and electric field intensity. Mafa Takisa and Maharaj [47] obtained new exact solutions to the Einstein-Maxwell system of equations with a polytropic equation of state. Thirukkanesh and Ragel [48] have obtained particular models of anisotropic fluids with polytropic equation of state consistent with the reported experimental observations. Malaver [49] generated new exact solutions to the Einstein-Maxwell system considering Van der Waals's modified equation of state with polytropic exponent. Bhar and Murad [50] obtained new relativistic stellar models with a particular type of metric function and a generalized Chaplygin equation of state. Recently Tello-Ortiz et al. [51] also found an anisotropic fluid sphere solution of the Einstein-Maxwell field equations with a modified version of the Chaplygin equation.

Presently there are efforts underway to understand the underlying quantum aspects with astrophysical-charged stellar models [52-57]. How the energy matter quantum wavefunction creates situations with the equation of state potential, expansions with quintessence field cosmologies with interior having dark energy matter generation compact stellar anisotropic gravitational potential and structure of many objects, especially strange quark stars as well have been key in Quantum Astrophysical projects ongoing [54-60]. There is also a study of the symmetry group theory with authors advancing that will help to classify general field-particle metrics linking towards Standard Model Particle Physics String Theories with Hubble and James Webb Telescope observations of the expanding universe models that are supposed to manifest from natural astrophysical Big Bang Theory [56-64].

It is important to mention the fact that general relativity not only studies the interior of stellar objects, it also allows the analysis of different cosmological scenarios through Einstein's gravity theory as the existence of dark energy, dark matter, Phantom and Quintessence fields that were introduced to explain the accelerated expansion of the universe [51,65]. Chaplygin gas whose equation of state $P=-\frac{B}{\rho}$ where p is the pressure, ρ the energy-density and B a positive constant, has been considered an alternative to the Phantom and Quintessence fields [50-66]. In order to adjust this equation of state to observational data has been rewritten as $P=-\frac{B}{\rho^{\omega}}$ with the parameter ω between 0 and 1 [66]. Furthermore, an extended version of the Chaplygin gas equation of state was proposed by Pourhassan [67] and its form is $P = A\rho - \frac{B}{\rho^{\omega}}$ where A a positive parameter constrained to $0 < A < 1/3$.

In this paper, we generated a new model of a charged anisotropic compact object with the modified Chaplygin equation of state proposed for Pourhassan [67] and studied by Bernardini and Bertolami [68]. The modified Chaplygin equation of state is described by $P = A\rho - \frac{B}{\rho^{\alpha}}$ where A, B, α are constants and $0 \le \alpha \le 1$. If we take α=1 then it gives generalized Chaplygin equation of state [50]. Using a particular form of gravitational potential $Z(x)$ that is nonsingular, continuous and well behaved in the interior of the star, we can obtain a new class of static spherically symmetrical model for a charged anisotropic matter distribution. It is expected that the solution obtained in this work can be applied in the description and the study of the internal structure of strange quark stars. The article is organized as follows: In section 2 we present Einstein-Maxwell field equations. In section 3 we make a particular choice for gravitational potential $Z(x)$ and the electric field intensity and generate new models for charged anisotropic matter. In Section 4, physical acceptability conditions are discussed. The physical properties and physical validity of these new solutions are analyzed in section 5. The conclusions of the results obtained are shown in section 6.

2. Einstein-Maxwell system of equations

We consider a spherically symmetric, static and homogeneous space-time. In Schwarzschild coordinates, the metric is given by:

$$ds^2 = -e^{2\nu(r)}dt^2 + e^{2\lambda(r)}dr^2 + r^2(d\theta^2 + \sin^2\theta d\varphi^2) \tag{1}$$

where $\nu(r)$ and $\lambda(r)$ are two arbitrary functions.

The Einstein field equations for the charged anisotropic matter are given by [30]:

$$\frac{1}{r^2}\left(1-e^{-2\lambda}\right) + \frac{2\lambda'}{r}e^{-2\lambda} = \rho + \frac{1}{2}E^2 \tag{2}$$

$$-\frac{1}{r^2}\left(1-e^{-2\lambda}\right) + \frac{2\nu'}{r}e^{-2\lambda} = p_r - \frac{1}{2}E^2 \tag{3}$$

$$e^{-2\lambda}\left(\nu'' + \nu'^2 + \frac{\nu'}{r} - \nu'\lambda' - \frac{\lambda'}{r}\right) = p_t + \frac{1}{2}E^2 \tag{4}$$

$$\sigma = \frac{1}{r^2}e^{-\lambda}(r^2E)' \tag{5}$$

where ρ is the energy density, p_r is the radial pressure, E is electric field intensity, p_t is the tangential pressure and primes denote differentiations concerning r. Using the transformations $x = Cr^2$, $Z(x) = e^{-2\lambda(r)}$ and $A_*^2 y^2(x) = e^{-2\nu(r)}$ with arbitrary constants A_* and C>0 suggested by Durgapal and Bannerji [69], the Einstein field equations can be written as:

$$\frac{1-Z}{x} - 2\dot{Z} = \frac{\rho}{C} + \frac{E^2}{2C} \tag{6}$$

$$4Z\frac{\dot{y}}{y}-\frac{1-Z}{x}=\frac{p_r}{C}-\frac{E^2}{2C} \quad (7)$$

$$4xZ\frac{\ddot{y}}{y}+(4Z+2x\dot{Z})\frac{\dot{y}}{y}+\dot{Z}=\frac{p_t}{C}+\frac{E^2}{2C} \quad (8)$$

$$p_t=p_r+\Delta \quad (9)$$

$$\frac{\Delta}{C}=4xZ\frac{\ddot{y}}{y}+\dot{Z}\left(1+2x\frac{\dot{y}}{y}\right)+\frac{1-Z}{x}-\frac{E^2}{C} \quad (10)$$

$$\sigma^2=\frac{4CZ}{x}\left(x\dot{E}+E\right)^2 \quad (11)$$

σ is the charge density, $\Delta=p_t-p_r$ is the anisotropy factor and dots denote differentiations concerning x.With the transformations of [69], the mass within a radius r of the sphere takes the form:

$$M(x)=\frac{1}{4C^{3/2}}\int_0^x\sqrt{x}\left(\rho^*+E^2\right)dx \quad (12)$$

Where

$$\rho^*=\left(\frac{1-Z}{x}-2\dot{Z}\right)C$$

In this paper, we assume the following equation of state where the radial pressure and the density ρ are related to the following form:

$$p_r=A\rho-\frac{B}{\rho} \quad (13)$$

with A and B as constant parameters, and α=1.
with A and B as constant parameters and $\rho=\rho*+E^2$.

3. Charged Anisotropic Model

In this work, we take the form of the gravitational potential *Z(x)* as *Z(x)=1-ax* proposed for Malaver [38] and Thirukanesh and Ragel [48] where a is a real constant. This potential is regular at the origin and well behaved in the interior of the sphere. Following Liguda et al. [70] for the electric field, we make the particular choice:

$$\frac{E^2}{2C}=kxZ(x)=kx(1-ax) \quad (14)$$

This electric field is finite at the center of the star and remains continuous in the interior. Using $Z(x)$ and eq. (14) in eq. (6), we obtain:

$$\rho=C\left[3a-kx\left(1-ax\right)\right] \quad (15)$$

Substituting eq. (15) in eq. (13), the radial pressure can be written in the form:

$$p_r = AC\left[3a - kx(1-ax)\right] - \frac{B}{C\left[3a - kx(1-ax)\right]} \tag{16}$$

Using eq. (15) in eq. (12), the expression of the mass function is

$$M(x) = \frac{\left(5akx^2 - 7kx + 35a\right)x^{3/2}}{70\sqrt{c}} \tag{17}$$

With eq. (14) and *Z(x)* in eq. (11), the charge density is

$$\sigma^2 = 2C^2k\left(3-4ax\right)^2 \tag{18}$$

With equations (13), (14), (15) and $Z(x)$, eq. (7) becomes:

$$\frac{\dot{y}}{y} = \frac{A\left[3a - kx(1-ax)\right]}{4(1-ax)} - \frac{B}{4C^2(1-ax)\left[3a - kx(1-ax)\right]} + \frac{a}{4(1-ax)} - \frac{kx}{4} \tag{19}$$

Integrating eq. (19) we obtain:

$$y(x) = c_1\left(akx^2 - kx + 3a\right)^{C*}\left(ax-1\right)^{D} e^{-\frac{Ex^2 + 2B\arctan\left(\frac{2akx-k}{\sqrt{12a^2k-k^2}}\right)}{24a^2C^2\sqrt{12a^2k-k^2}}} \tag{20}$$

where for convenience we have let

$$C* = -\frac{B}{24C^2a^2} \tag{21}$$

$$D = -\frac{3}{4}A - \frac{1}{4} + \frac{B}{12C^2a^2} \tag{22}$$

$$E = 3k\left(A+1\right)a^2C^2\sqrt{12a^2k-k^2} \tag{23}$$

and c_1 is the constant of integration.

The metric functions $e^{2\lambda}$ and $e^{2\nu}$ can be written as:

$$e^{2\lambda} = \frac{1}{1-ax} \tag{24}$$

$$e^{2\nu} = c_1^2\left(akx^2 - kx + 3a\right)^{2C*}\left(ax-1\right)^{2D} e^{-\frac{Ex^2 + 2B\arctan\left(\frac{2akx-k}{\sqrt{12a^2k-k^2}}\right)}{12a^2C^2\sqrt{12a^2k-k^2}}} \tag{25}$$

and the anisotropy Δ is given by:

$$\Delta = 4xC(1-ax)\left[\begin{array}{l}\dfrac{C*^2(2akx-k)^2}{\left(akx^2-kx+3a\right)^2}+\dfrac{2C*ak}{akx^2-kx+3a}-\dfrac{C*(2akx-k)^2}{\left(akx^2-kx+3a\right)^2}\\ +\dfrac{2C*Da(2akx-k)}{\left(akx^2-kx+3a\right)(ax-1)}-\dfrac{C*(2akx-k)}{12a^2\left(akx^2-kx+3a\right)\sqrt{12a^2k-k^2}}\left(2Ex+\dfrac{4Bak}{\sqrt{12a^2k-k^2}\left(1+\dfrac{(2akx-k)^2}{12a^2k-k^2}\right)}\right)\\ \dfrac{\left(D^2-D\right)a^2}{(ax-1)^2}-\dfrac{D}{12a(ax-1)\sqrt{12a^2k-k^2}}\left(2Ex+\dfrac{4Bak}{\sqrt{12a^2k-k^2}\left(1+\dfrac{(2akx-k)^2}{12a^2k-k^2}\right)}\right)\\ -\dfrac{\left(2E-\dfrac{16Ba^2k^2(2akx-k)}{\left(12a^2k-k^2\right)^{3/2}\left(1+\dfrac{(2akx-k)^2}{12a^2k-k^2}\right)^2}\right)}{24a^2\sqrt{12a^2k-k^2}}+\dfrac{\left(2Ex+\dfrac{4Bak}{\sqrt{12a^2k-k^2}\left(1+\dfrac{(2akx-k)^2}{12a^2k-k^2}\right)}\right)^2}{576a^4\left(12a^2k-k^2\right)}\end{array}\right]$$

$$-2xaC\left[\frac{2xC*(2akx-k)}{akx^2-kx+3a}+\frac{Da}{ax-1}-\frac{\left(2Ex+\dfrac{4Bak}{\sqrt{12a^2k-k^2}\left(1+\dfrac{(2akx-k)^2}{12a^2k-k^2}\right)}\right)}{24a^2\sqrt{12a^2k-k^2}}\right]-2kxC(1-ax) \qquad (26)$$

4. Physical Requirements for the New Model

Following Delgaty and Lake [3], Thirukkanesh and Ragel [48] and Bibi et al. [71] for a model to be physically acceptable, it must satisfy the following conditions:

(i). Regularity of the metric potentials in the stellar interior and at the origin.

(ii). The radial pressure should be positive, decreasing with the radial coordinate and vanishing at the center of the fluid sphere.

(iii). The energy density should be positive inside of the star and a decreasing function of the radial parameter.

(iv). The radial pressure and density gradients $dp_r/dr \le 0$ and $d\rho/dr \le 0$ for $0 \le r \le R$.

(v). The causality condition requires that the radial speed of sound should be less than the speed of light throughout the model, i.e. $0 \le \frac{dp_r}{d\rho} \le 1$.

(vi). The radial pressure and the anisotropy are equal to zero at the center of the fluid sphere $\Delta(r=0)=0$.

(vii). The charged interior solution should be matched with the Reissner–Nordström exterior solution, for which the metric is given by:

$$ds^2 = -\left(1-\frac{2M}{r}+\frac{Q^2}{r^2}\right)dt^2 + \left(1-\frac{2M}{r}+\frac{Q^2}{r^2}\right)^{-1} dr^2 + r^2\left(d\theta^2 + \sin^2 d\varphi^2\right) \tag{27}$$

Through the boundary $r=R$ where M and Q are the total mass and the total charge of the star, respectively.

Conditions (ii), (iii) and (iv) imply that the radial pressure and energy density must reach a maximum at the center and decreasing towards the surface of the sphere.

5. Physical Analysis

We now present the analysis of the physical characteristics of the new model. The metric functions $e^{2\lambda}$ and $e^{2\nu}$ should remain positive throughout the stellar interior and in the origin $e^{2\lambda(0)}=1$, $e^{2\nu(0)}=c_1^2\left(3a\right)^{2C*}\left(-1\right)^{2D} e^{\frac{B\arctan\left(\frac{k}{\sqrt{12a^2k-k^2}}\right)}{6a^2C^2\sqrt{12a^2k-k^2}}}$. We note in $r=0$ that $\left(e^{2\lambda(r)}\right)'_{r=0} = \left(e^{2\nu(r)}\right)'_{r=0} = 0$. This demonstrates that the gravitational potentials are regular at the center $r=0$. The energy density and radial pressure are positive and well behaved inside the stellar interior. Also, we have the central density and pressure $\rho(0)=3aC$, $p_r(0)=3AaC-\frac{B}{3aC}$. According to the expression of radial pressure, $p_r(0)$ will be non-negative at the center as it is satisfied by condition $3AaC > \frac{B}{3aC}$.

In the surface of the star $r=R$, we have $p_r(r=R)=0$ and

$$R = \frac{\sqrt{2ak\left(k+\sqrt{4ak\sqrt{\frac{B}{A}}-12a^2k+k^2}\right)}}{2ak} \tag{28}$$

For a realistic star, it is expected that the gradient of energy density and radial pressure should be decreasing functions of the radial coordinate r. In this model, for all $0 < r < R$, we obtain respectively:

$$\frac{d\rho}{dr} = -2kCr^2\left(1-aCr^2\right)+2kC^3r^3a < 0 \tag{29}$$

$$\frac{dp_r}{dr} = AC\left[-2kCr\left(1-aCr^2\right)+2kC^2r^3a\right] + \frac{BC\left[-2kCr\left(1-aCr^2\right)+2kC^2r^3a\right]}{\left[3a-kCr^2\left(1-aCr^2\right)\right]^2} < 0 \tag{30}$$

and according to the equations (29) and (30), the energy density and radial pressure decrease from the center to the surface of the star.

From equation (17), we have for the total mass of the star:

$$M(r=R)=\frac{\left(5akC^2R^4-7kCR^2+35a\right)CR^3}{70} \tag{31}$$

The causality condition demands that the radial sound speed defined as $v_{sr}^2=\frac{dp_r}{d\rho}$ should not exceed the speed of light and it must be within the limit $0\le v_{sr}^2\le 1$ in the interior of the star [3]. With the transformations of Durgapal and Bannerji [69] in this model we have:

$$0\le v_{sr}^2=A+\frac{B}{C^2\left[3a-kCr^2\left(1-aCr^2\right)\right]^2}\le 1 \tag{32}$$

On the boundary $r=R$, the solution must match the Reissner–Nordström exterior space–time as:

$$ds^2=-\left(1-\frac{2M}{r}+\frac{Q^2}{r^2}\right)dt^2+\left(1-\frac{2M}{r}+\frac{Q^2}{r^2}\right)^{-1}dr^2+r^2\left(d\theta^2+\sin^2 d\varphi^2\right) \tag{33}$$

and therefore, the continuity of e^{ν} and e^{λ} across the boundary $r=R$ is

$$e^{2\nu}=e^{-2\lambda}=1-\frac{2M}{R}+\frac{Q^2}{R^2} \tag{34}$$

Then for the matching conditions, we obtain:

$$\frac{2M}{R}=aCR^2+2kC^2R^4-2akC^3R^6 \tag{35}$$

Table 1 presents the values of the parameters chosen *K*, *A*, *B* and *a*. The masses of stellar objects are also shown

Table 1. Parameters a, A, B and stellar masses for different values of k

k	A	$B(x10^{-5})$	a	$M(M_{\Theta})$
0.0011	0.2	1.5	0.011	$0.60M_{\Theta}$
0.0012	0.2	1.5	0.011	$0.55M_{\Theta}$
0.0013	0.2	1.5	0.011	$0.48M_{\Theta}$

Where M_{Θ} is the mass of the sun.

Figures 1, 2, 3, 4, 5, 6, 7, 8 and 9 represent the graphs of ρ, p_r, M, σ^2, $\frac{E^2}{2C}$, Δ, v_{sr}^2, $\frac{d\rho}{dr}$ and $\frac{dp_r}{dr}$ with the radial coordinate, respectively. In all the cases we have considered *C=1*.

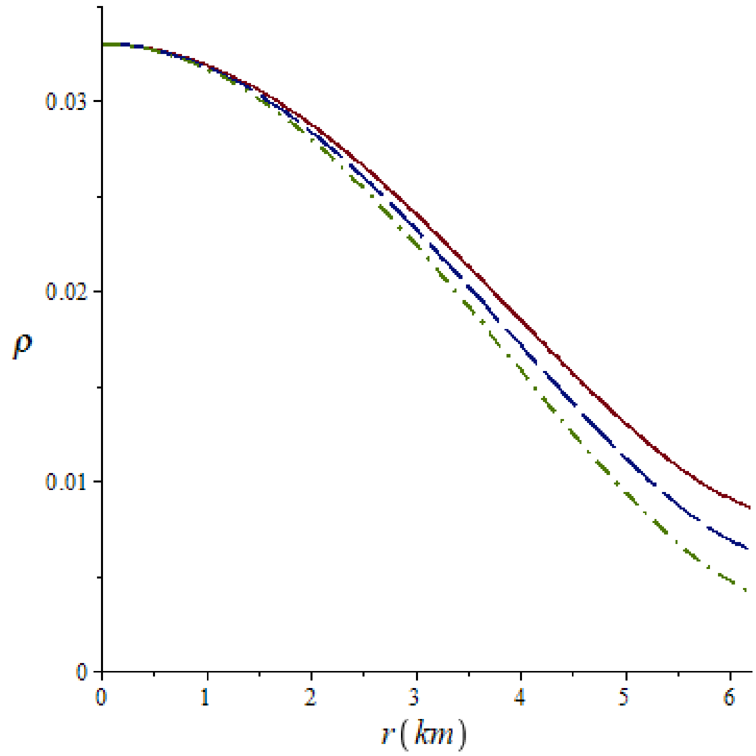

Figure 1. Variation of energy density with the radial coordinate for k=0.0011 (solid line), k=0.0012 (long-dash line) and k=0.0013 (dash-dot line).

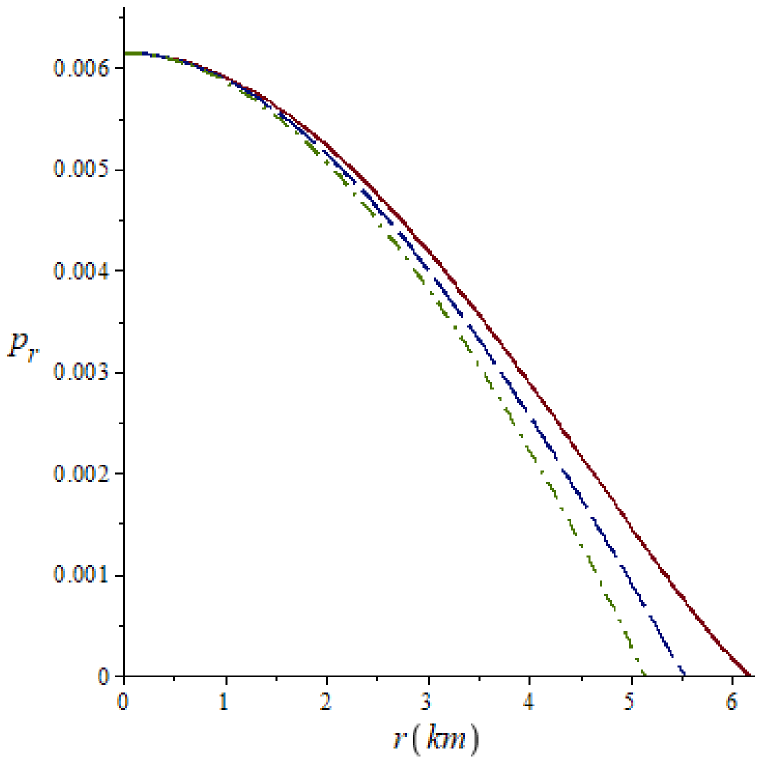

Figure 2. Variation of radial pressure with the radial coordinate for k=0.0011 (solid line), k=0.0012 (long-dash line) and k=0.0013 (dash-dot line).

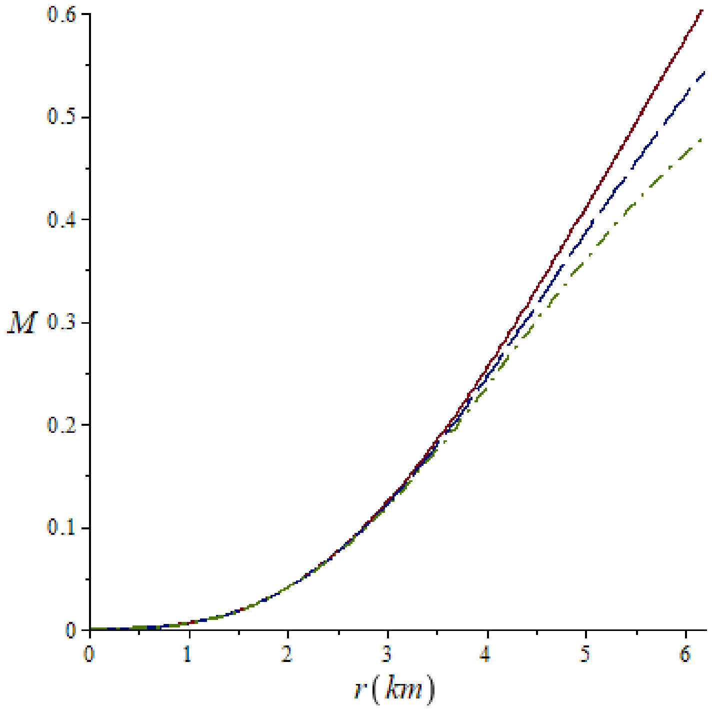

Figure 3. Variation of Mass function *M* with the radial parameter for k=0.0011 (solid line), k=0.0012 (long-dash line) and k=0.0013 (dash-dot line).

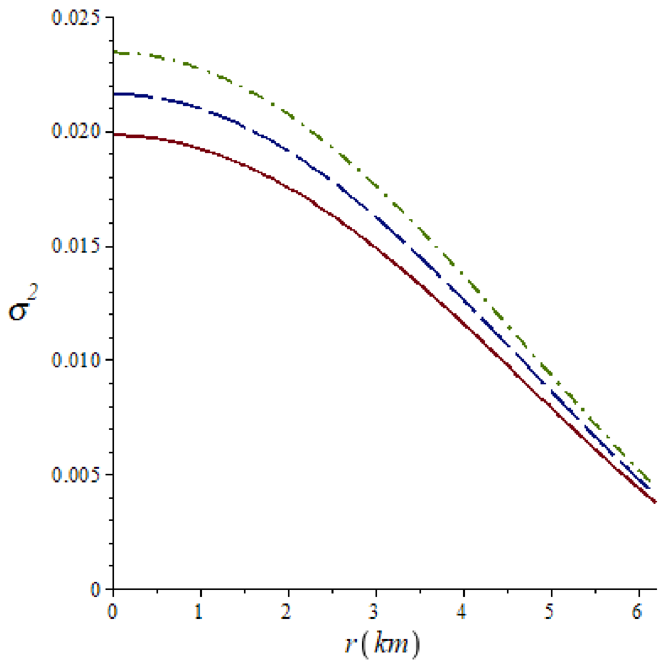

Figure 4. Variation of charge density σ^2 with the radial parameter for k=0.0011 (solid line), k=0.0012 (long-dash line) and k=0.0013 (dash-dot line).

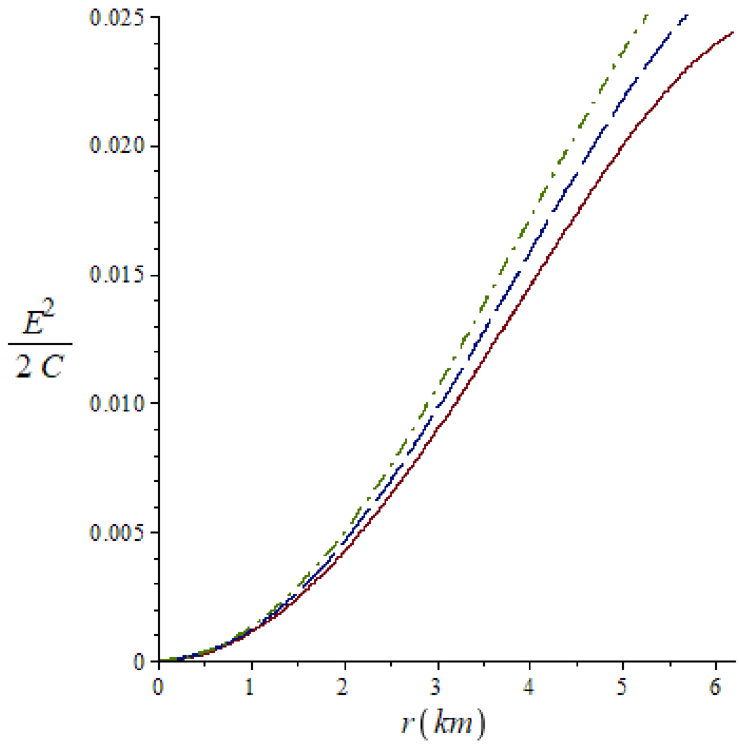

Figure 5. Variation of electric field intensity with the radial parameter for k=0.0011 (solid line), k=0.0012 (long-dash line) and k=0.0013 (dash-dot line).

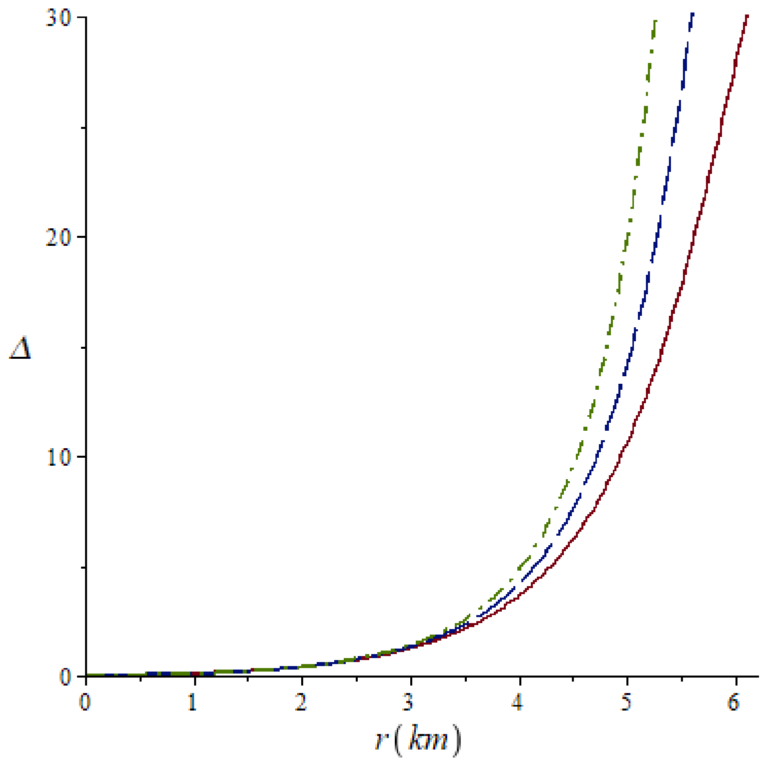

Figure 6. Variation of anisotropy with the radial parameter for k=0.0011 (solid line), k=0.0012 (long-dash line) and k=0.0013 (dash-dot line).

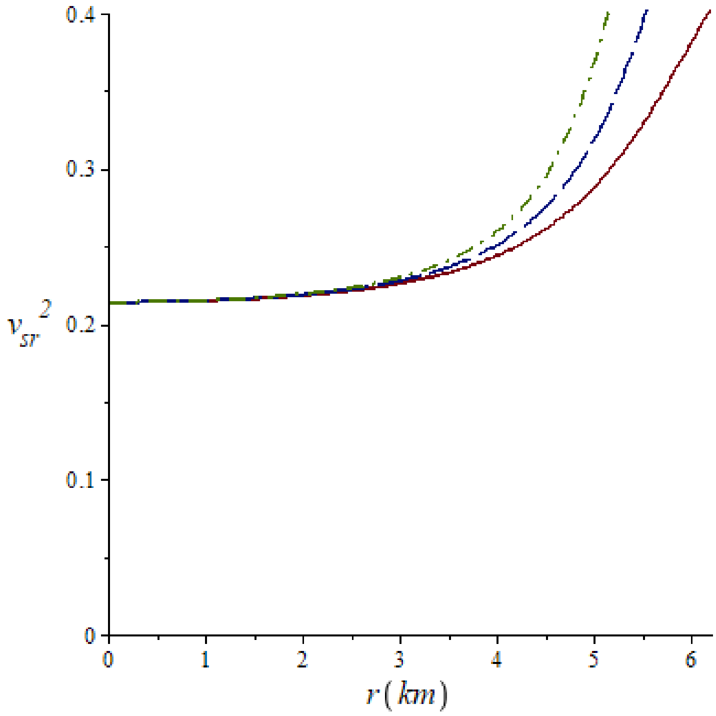

Figure 7. Variation of radial speed sound with a radial coordinate for k=0.0011 (solid line), k=0.0012 (long-dash line) and k=0.0013 (dash-dot line).

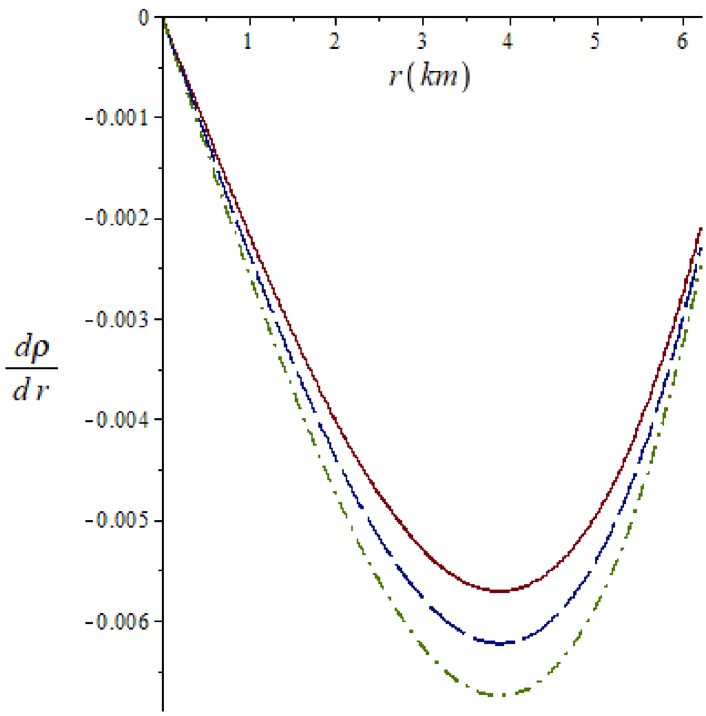

Figure 8. Variation of the gradient of density with a radial coordinate for k=0.0011 (solid line), k=0.0012 (long-dash line) and k=0.0013 (dash-dot line).

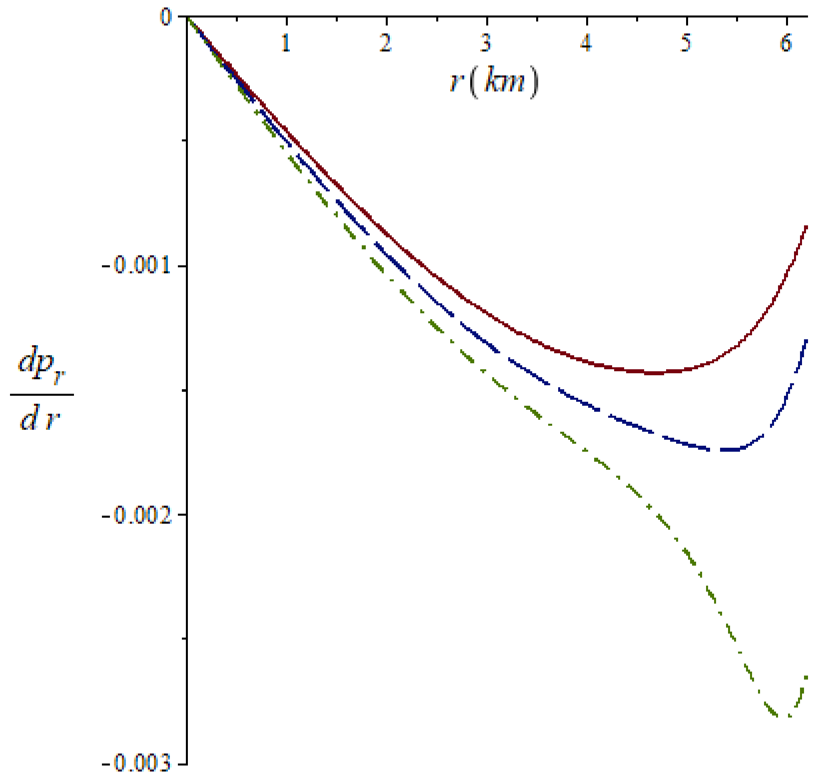

Figure 9. Variation of the gradient of radial pressure with a radial coordinate for k=0.0011 (solid line), k=0.0012 (long-dash line) and k=0.0013 (dash-dot line).

Figure 1 shows that the energy density is continuous, finite, decreases radially outward and vanishes at the boundary. In Figure 2, we note that the radial pressure ρ also is finite, continuous and monotonically decreasing function. In Figure 3, it is observed that the mass function is regular, strictly increasing and well behaved. Figure 4 shows that the charge density is regular at the center, non-negative and decreases with the radial parameter for the chosen k values. In Figure 5, the electric field intensity E^2 is positive and monotonically increasing throughout the interior of the star in all the considered cases. In Figure 6, the anisotropy factor Δ vanishes at $r=0$, it monotonically increases and is continuous in the stellar interior. In Figure 7, we note that the $v_{sr}^2 = \frac{dp_r}{d\rho}$ is within the desired range $0 \le v_{sr}^2 \le 1$ for the different values of k, which is a physical requirement for the construction of a realistic star [3]. Figures 8 and 9 respectively show that the gradients of radial pressure $\frac{d\rho}{dr}$ and energy density $\frac{dp_r}{dr}$ are decreasing throughout the star.

We can compare the values calculated for the mass function with observational data. For k=0.0011 the values of A, B and a allow us to obtain a mass of 0.6$M_\odot$ which can correspond to astronomic object GJ 440 also known as LHS 43 [72], or could be associated with the orange dwarf GJ 380 [73]. For the case k=0.0012 we obtained comparable masses with the red dwarf Lacaille 8760 with a mass between (0.56-0.60) $M_\odot$ [74]. With k=0.0013, the resulting mass is very similar to the red dwarf Lalande 21185 whose mass is 0.46 $M_\odot$ [75]. The values of the masses for these compact stars are tabulated in Table 2.

Table 2. The reported values of the masses for the compact stars

Compact Star	Masses $M(M_\Theta)$
LHS 43	$0.62M_\Theta$
GJ 380	$0.64M_\Theta$
Lacaille 8760	$(0.56\text{-}0.60)M_\Theta$
Lalande 21185	$0.46M_\Theta$

There is also a quantum contribution to these masses, since the state of the clock affects environment vacuum oscillations, like neutrino oscillations that change the flavor of the quark-gluon-plasma as well as switching quaternion operation of gauge fields of light as well as sound outputs quantum activities [8, 52, 54, 61, 62]. The underlying mass effects on dwarf compact stars perhaps will explain their variability with energy density, pressure, mass function, charge density, anisotropy, the electric intensity of field, especially in the interior of these stellar objects, and radial sound aspects correlating results demonstrated successfully above [52, 55, 56, 57, 58, 59, 60, 62, 63, 64].

6. Conclusion

In this work we have developed some simple relativistic charged stellar models obtained by solving Einstein-Maxwell field equations for a static spherically symmetric locally anisotropic fluid distribution. By choice of metric potential and electrical charge distribution, together with the Chaplygin equation of state the behavior of fluid distribution has been studied. With the positive anisotropy, $p_t > p_r$, the stability of the new solutions is examined by the condition $0 \le v_{sr}^2 \le 1$ and it is found that the model developed is potentially stable for the parameters considered. An analytical stellar model with such physical features could play a significant role in the description of internal structure of electrically charged strange stars. The newly obtained models match smoothly with the Schwarzschild exterior metric across the boundary $r=R$ because matter variables and the gravitational potentials of this research are consistent with the physical analysis of these stars.

The new solutions can be related to stellar objects such as LHS 43, GJ 380, Lacaille 8760 and Lalande 21185. Physical features associated with the matter, radial pressure, density, anisotropy, charge density and the plots generated suggest that the model with $k=0.0013$, is similar to the red dwarf Lalande 21185 is well behaved [75]. We have ansatz formalisms that connect astrophysics with the quantum nature of these anisotropic matter in stellar compact objects, with observable parameters derived from theoretical modeling to experimental measurements. These have all been necessitated by especially current findings of the James Webb Telescope of six earlier formed massive galaxies to peek into quantum nature with our newly developed point-to-point signal/noise matrix measurements of vibrational or sound and photonic or light gauge fields.

How the energy matter wavefunction creates situations with the equation of state potential, expansion with quintessence field cosmologies with interior having dark energy matter generation compact stellar anisotropic gravitational potential and structure of many objects, especially strange quark stars as well have been key in Quantum Astrophysical projects ongoing. The underlying mass effects on dwarf compact stars perhaps will explain their variability with energy density, pressure, mass function, charge density, anisotropy, electric intensity of the field, especially in the interior of these stellar objects, and radial sound aspects correlating results demonstrated successfully above. quantum contribution to these masses, since the state_of_the_clock affects environment vacuum oscillations, like neutrino oscillations that change the flavor of the quark-gluon-plasma as well as switching quaternion operation of gauge fields of light as well as sound outputs quantum activities.

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